Handout 13

MATH 140 Lab: Section 1

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Note: *This handout gives you an introduction about* ***Separable Method*** *in Differential Equations.*

All Examples in this handout are taken from my recent published textbook:

A Friendly Introduction to Differential Equations, *Kaabar, M, Vol 1,164, Jan. 05, 2015.*

If you want to learn more about differential equations because you may have a question in exam 2 or final exam about differential equations, I encourage you to download and read sections 1.1 and 4.2 in my textbook.

Instructions to download my textbook for free:

1- Please go to (http://www.mohammed-kaabar.net/#!differential-equations-book/cuvt)
2- Then, go to "**Links to view and download this book**"
3- Choose either "**ResearchGate PDF Format**" or "**Google Books (100% viewable for free**".

Separable Method

We will solve some differential equations using a method known as Separable Method. This method is called separable because we separate two different terms from each other.

Definition 1 The standard form of Separable Method is written as follows:

$$(All\ in\ terms\ of\ x)dx - (All\ in\ terms\ of\ y)dy = 0$$

Note: it does not matter whether it is the above form or in the following form:

$$(All\ in\ terms\ of\ y)dy - (All\ in\ terms\ of\ x)dx = 0$$

Example 1: Solve the following differential equation: $\frac{dy}{dx} = \frac{y^3}{(x+3)}$

Solution: By using definition 1 , we need to rewrite the above equation in a way that each term is separated from the other term as follows:

$$\frac{dy}{dx} = \frac{y^3}{(x+3)} = \frac{\frac{1}{(x+3)}}{\frac{1}{y^3}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Now, we need to do a cross multiplication for (1) as follows:

$$\frac{1}{y^3}dy = \frac{1}{(x+3)}dx$$

$$\frac{1}{y^3}dy - \frac{1}{(x+3)}dx = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Then, we integrate both sides of (2) as follows:

$$\int \left(\frac{1}{y^3}dy - \frac{1}{(x+3)}dx\right) = \int 0$$

$$\int \left(\frac{1}{y^3}\right)dy - \int \left(\frac{1}{(x+3)}\right)dx = c$$

$$\int (y^{-3})dy - \int \left(\frac{1}{(x+3)}\right)dx = c$$

$$-\frac{1}{2}y^{-2} - \ln(|(x+3)|) = c$$

Thus, the general solution is :

$$-\frac{1}{2}y^{-2} - \ln(|(x+3)|) = c$$

Example 2: Solve the following differential equation: $\frac{dy}{dx} = e^{3y+2x}$

Solution: By using definition 1 , we need to rewrite the above equation in a way that each term is separated from the other term as follows:

$$\frac{dy}{dx} = e^{3y+2x} = e^{3y} \cdot e^{2x} = \frac{e^{2x}}{e^{-3y}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Now, we need to do a cross multiplication for (1) as follows:

$$e^{-3y}dy = e^{2x}dx$$

$$e^{-3y}dy - e^{2x}dx = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Then, we integrate both sides of (2) as follows:

$$\int (e^{-3y}dy - e^{2x}dx) = \int 0$$

$$\int (e^{-3y})dy - \int (e^{2x})\,dx = c$$

$$-\frac{1}{3}e^{-3y} - \frac{1}{2}e^{2x} = c$$

Thus, the general solution is :

$$-\frac{1}{3}e^{-3y} - \frac{1}{2}e^{2x} = c$$

Separable Method is Awesome

Best Regards,

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