Handout 7

MATH 172 Lab: Sections 7 and 8

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Note: *This handout is a review for exam 2 in MATH 172.*

- **Useful Integrals and Identities**

$$\int \sec^2(x)dx = \tan(x) + C \quad \bullet \quad \int \sec(x)\tan(x)dx = \sec(x) + C$$

$$\int \cosh(x)dx = \sinh(x) + C \quad \bullet \quad \int \sinh(x)dx = \cosh(x) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \bullet \quad \int \frac{1}{x^2 + a^2}dx = \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \cot(x)dx = \ln|\sin(x)| + C \quad \bullet \quad \int \tan(x)dx = \ln|\sec(x)| + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C \quad \bullet \quad \int \frac{1}{x}dx = \ln|x| + C$$

$$\int \tan(x)dx = \ln|\sec(x)| + C \quad \bullet \quad \int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x)) \quad \bullet \quad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) + \cos^2(x) = 1 \quad \bullet \quad 1 - \sin^2(x) = \cos^2(x)$$

$$1 + \tan^2(x) = \sec^2(x) \quad \bullet \quad \sec^2(x) - 1 = \tan^2(x)$$

- **Definitions**

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- **Derivatives**

$$\frac{d}{dx}[\sinh(x)] = \cosh(x)$$

$$\frac{d}{dx}[\cosh(x)] = \sinh(x)$$

$$\frac{d}{dx}[\tanh(x)] = \text{sech}^2(x)$$

- Integrals of inverse hyperbolics

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1}(x) + C$$
$$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1}(x) + C \qquad x > 1$$
$$\int \frac{dx}{1-x^2} = \tanh^{-1}(x) + C \qquad |x| < 1$$

- **Examples from my textbooks in differential equations and linear algebra for Integration by Parts and Partial Fractions using <u>Table Method</u> and <u>Cover Method</u>:**

Example 1:

Example 1.2.4 Find $\int 3x^2 \sin(4x)dx$.

Solution: To find $\int 3x^2 \sin(4x)dx$, it is easier to use the table method than using integration by parts. In the table method, we need to create two columns: one for derivatives of $3x^2$, and the other one for integrations of $\sin(4x)$. Then, we need to keep deriving $3x^2$ till we get zero, and we stop integrating when the corresponding row is zero. The following table shows the table method to find $\int 3x^2 \sin(4x)dx$:

Derivatives Part	Integration Part
$3x^2$	$\sin(4x)$
$6x$	$-\frac{1}{4}\cos(4x)$ +
6	$-\frac{1}{16}\sin(4x)$ −
0	$\frac{1}{64}\cos(4x)$ +

Table 1.2.2: Table Method for $\int 3x^2 \sin(4x)dx$

We always start with positive sign, followed by negative sign, and so on as we can see in the above table 1.2.2. Now, from the above table 1.2.2, we can find $\int 3x^2 \sin(4x)dx$ as follows:

$$\int 3x^2 \sin(4x)dx$$
$$= -\frac{1}{4}(3)x^2\cos(4x) - \left(-\frac{1}{16}\right)6x\sin(4x) + \left(\frac{1}{64}\right)6\cos(4x) + C$$

Thus, $\int 3x^2 \sin(4x)dx = -\frac{3}{4}x^2\cos(4x) + \frac{3}{8}x\sin(4x) + \frac{3}{32}\cos(4x) + C$.

Example 2:

Since the numerator has a polynomial of degree 0 ($x^0 = 1$), and the denominator a polynomial of degree 2, then this means the degree of numerator is less than the degree of denominator. Thus, in this case, we need to use the partial fraction as follows:

$$\frac{1}{(s-2)(s+2)} = \frac{a}{(s-2)} + \frac{b}{(s+2)}$$

It is easier to use a method known as cover method than using the traditional method that takes long time to finish it. In the cover method, we cover the original, say $(s-2)$, and substitute $s = 2$ in $\frac{1}{(s-2)(s+2)}$ to find the value of a. Then, we cover the original, say $(s+2)$, and substitute $s = -2$ in $\frac{1}{(s-2)(s+2)}$ to find the value of b.

Thus, $a = \frac{1}{4}$ and $b = -\frac{1}{4}$. This implies that

$$\frac{1}{(s-2)(s+2)} = \frac{\frac{1}{4}}{(s-2)} + \frac{-\frac{1}{4}}{(s+2)}$$

Example 3:

Since the numerator has a polynomial of degree 0 ($x^0 = 1$), and the denominator a polynomial of degree 3, then this means the degree of numerator is less than the degree of denominator. Thus, in this case, we need to use the partial fraction as follows:

$$\frac{1}{s(s+3)(s+2)} = \frac{a}{s} + \frac{b}{(s+3)} + \frac{c}{(s+2)}$$

Now, we use the cover method. In the cover method, we cover the original, say s, and substitute $s = 0$ in $\frac{1}{s(s+3)(s+2)}$ to find the value of a. We cover the original,

say $(s+3)$, and substitute $s = -3$ in $\frac{1}{s(s+3)(s+2)}$ to find the value of b. Then, we cover the original, say $(s+2)$, and substitute $s = -2$ in $\frac{1}{s(s+3)(s+2)}$ to find the value of c. Thus, $= \frac{1}{6}$, $b = \frac{1}{3}$ and $c = -\frac{1}{2}$. This implies that

$$\frac{1}{s(s+3)(s+2)} = \frac{\frac{1}{6}}{s} + \frac{\frac{1}{3}}{(s+3)} + \frac{-\frac{1}{2}}{(s+2)}$$

Good Luck in Exam 2

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