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Section: – Solution –

Parametric Equations Lab

Topics and skills: Parametric equations, graphing

Lissajous curves

Named after Jules Lissajous (1822-1880), **Lissajous curves** are generated by the parametric equations

$$x = A\cos at \qquad y = B\sin bt$$

where varying the amplitudes, A and B, and the frequencies, a and b, gives a huge variety of figures. Note that the only effect of A and B is to stretch or compress the figure in the x- and y-directions, respectively. Without much loss of generality, we take $A = B = 1$ and focus on the role of a and b, which we take to be integers.

1. Consider the equations $x = \cos 3t$, $y = \sin 2t$. Before using a graphing utility, it's advisable to find an interval for the parameter values that generates the complete curve. What is the period of $\cos 3t$? What is the period of $\sin 2t$? What is an interval which will generate this entire Lissajous curve, shown in Figure 1? Try to find the smallest one you can.

$[0, \pi]$ is the larger of the two intervals will not generate the whole function. $[0, \frac{2\pi}{3}]$ is the smallest interval which will generate the whole picture.

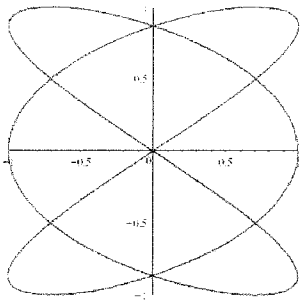


Figure 1

Solution:

Generally, the standard form: $A\cos(bx - c) + d$

① Amplitude $= |A|$

② Period $= \frac{2\pi}{|b|}$

③ Phase Shift $= \frac{c}{b}$

④ Vertical Shift $= d$

$(\cos(3t), \sin(2t))$ Domain $0 \le t \le 360$

So, me plot $x = \cos(3t)$ and $y = \sin(2t)$ using "Desmos.com".

The period of $x = \cos(3t)$ is $\frac{2\pi}{3}$ and the other one is $\frac{2\pi}{2} = \pi$.

The interval of $x = \cos(3t)$ is $[0, \frac{2\pi}{3}]$ and the other one is $[0, \pi]$.

2. Experiment with different values of a and b for the Lissajous curve. Write down at least three observations about how the graph changes as you vary a and b. You might consider the length of the interval needed for a complete picture, the shape of the graph, the number of bumps on each side of the graph, etc.

Possible Values

① $a=1, b=2$

② $a=3, b=2$

③ $a=3, b=4$

④ $a=5, b=4$

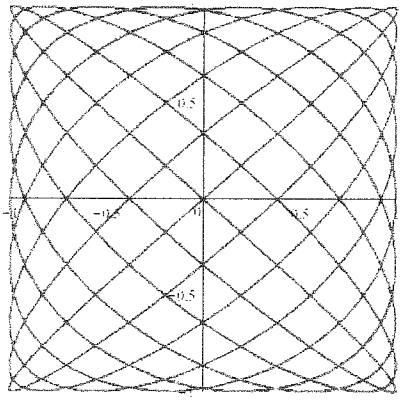
Any reasonable conclusion is correct with 3 or more observations or descriptions.

3. A nice family of Lissajous curves is generated by taking a to be an odd integer and $b = a \pm 1$. Match the following parametric equations with the curves in Figure 2.

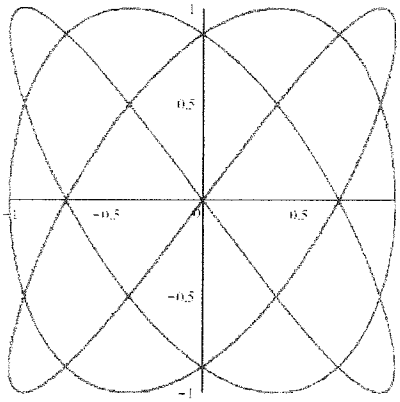
$\langle \cos(3t), \sin(4t) \rangle$
domain
$0 \le t \le 360$

A : $x = \cos 5t,\ y = \sin 4t$ B : $x = \cos 3t,\ y = \sin 4t$

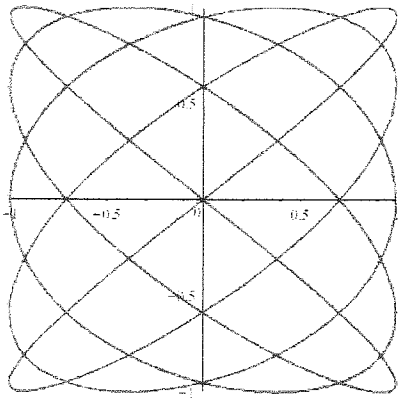
C : $x = \cos 5t,\ y = \sin 6t$ D : $x = \cos 9t,\ y = \sin 8t$



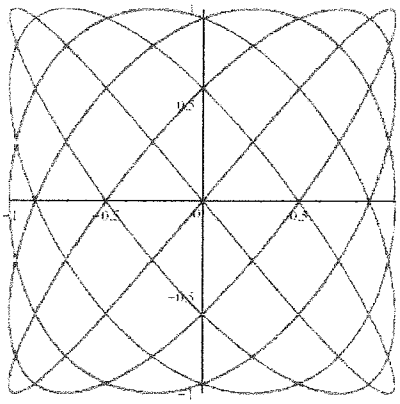
= D



= B



= A



= C

4. Without graphing, describe the curve produced by the equations $x = \cos 10t$, $y = \sin 10t$.

$x^2 = \cos^2(10t)$

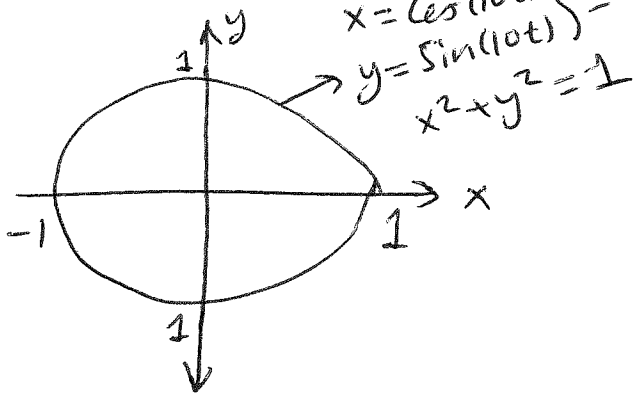
add + $y^2 = \sin^2(10t)$

$x^2 + y^2 = \cos^2(10t) + \sin^2(10t)$

$\Downarrow$ $\Downarrow$

$\boxed{x^2 + y^2 = 1}$

$0 \le t \le 2\pi$



Circle

↳ $(x-0)^2 + (y-0)^2 = 1^2$ circle centered at (0,0) and it has a radius of 1.

5. What is the effect of introducing a **phase angle** in the equations? For example, plot and describe the curves $x = \cos(5t + \pi/4)$, $y = \sin 4t$ and $x = \cos(5t)$, $y = \sin 4t$. What happens if you graph these parametric equations on a small interval like $[0, \pi]$?

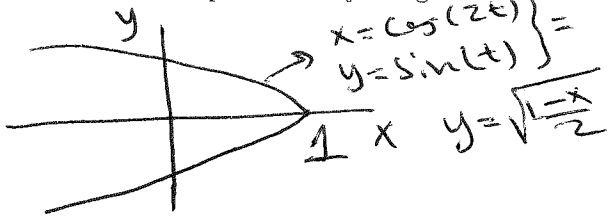
If you graph the entire picture, the two pictures will be the same. So, you have to graph a smaller interval to see the difference.

6. Graph the curve described by the equations $x = \cos 2t$, $y = \sin t$. Eliminate the parameter by using the trigonometric identity, $\cos 2t = 1 - 2\sin^2 t$.

$\boxed{x = \cos(2t) = 1 - 2\sin^2 t}$

$\boxed{y = \sin(t)}$

$\Downarrow$



$x = 1 - 2\sin^2 t = 1 - 2y^2$ since $y = \sin(t)$.

So, $\boxed{x = 1 - 2y^2}$

$\Rightarrow +2y^2 = 1 - x \Rightarrow y^2 = \frac{1-x}{2} \Rightarrow \boxed{y = \sqrt{\frac{1-x}{2}}}$