

**Problem Set 2 SOLUTIONS**

**Question 1:** If  $g(x) = x^2 - 3$  has a root in  $[1,3]$ . Use the bisection method to approximate the root that is accurate to at least within  $10^{-2}$ .

**SOLUTION**

**Step1:** we make sure that  $g(1)*g(3) < 0$  by finding both  $g(1)$  &  $g(3)$

$$g(1) = 1^2 - 3 = -2 < 0$$

$$g(3) = 3^2 - 3 = 9 - 3 = 6 > 0$$

Therefore,  $g(1)*g(3) = -12 < 0$

**Step2:** we create the following table:

n	$a_n$	$b_n$	$p_n$	$g(p_n)$
0	1	3	2	1
1	1	2	1.5	-0.75
2	1.5	2	1.75	0.0625
3	1.5	1.75	1.625	-0.359375
4	1.625	1.75	1.6875	-0.152343
5	1.6875	1.75	1.71875	-0.045898

**Question 2:** If  $g(x) = x^2 - 3$  has a root in  $[1,3]$ . How many iterations are needed for the bisection to get an error of less than  $10^{-2}$ .

**SOLUTION**

$$|E| \leq \left| \frac{b-a}{2^n} \right| < \text{tolerance}$$

$$\left| \frac{b-a}{2^n} \right| < \text{tolerance}$$

$$\left| \frac{3-1}{2^n} \right| < 10^{-2}$$

$$\left| \frac{2}{2^n} \right| < \frac{1}{10^2}$$

$$\left| \frac{1}{2^n} \right| < \frac{1}{2 \cdot 10^2}$$

$$2^n > 2 \cdot 10^2$$

$$\ln(2^n) > \ln(2 \cdot 10^2)$$

$$n \cdot \ln(2) > \ln(2 \cdot 10^2)$$

$$n > \frac{\ln(2 \cdot 10^2)}{\ln(2)} \approx 7.643$$

The number of iterations is 8 iterations.

**GOOD LUCK!**

**STUDY + LEARN + SOLVE QUIZ 2 = PASS QUIZ 2**

Best Regards

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