Quiz 4

MATH 172 Lab: Section 7

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Note: This quiz covers only the partial fractions and improper integrals.

Show your work and circle your answers. Neatness and organization count!

Question 1: (2 points) Decompose $\frac{5x^2+20x+6}{x^3+2x^2+x}$ into partial fractions. Be sure to find the values of any unknown constants.

$x^3+2x^2+x = x(x^2+2x+1) = x(x+1)^2 = x(x+1)(x+1)$

$$\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$$

$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

$x=0$: $\boxed{6=A}$

$x=-1$: $-9=-C \Rightarrow \boxed{C=9}$

$x=1$: $31 = 4A+2B+C$

$31 = 24+2B+9$

$-2 = 2B \Rightarrow \boxed{B=-1}$

The final solution is $\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}$

$$= \boxed{\frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}}$$

Question 2: (3 points) Compute the following improper integral by evaluating appropriate limits:

$$\int_{-\infty}^{0} xe^x dx$$

Hint: you may need to use integration by parts and L'Hôpital's rule.

$\int xe^x dx$ By parts

$u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$= \boxed{xe^x - e^x + C}$$

Now, $\int_{-\infty}^{0} xe^x dx = \lim_{t \to -\infty} \int_{t}^{0} xe^x dx$

$$= \lim_{t \to -\infty} \left[(0e^{(0)} - e^0) - te^t + e^t \right]$$

$$= \lim_{t \to -\infty} \left[-te^t + e^t \right]^0_t = -1 \text{ Convergent.}$$

OR another way: L'H $\lim_{t \to -\infty} te^t = \lim_{t \to -\infty} \frac{t}{e^{-t}} \overset{L'H}{=} \frac{1}{-e^{-t}}$

$$= \frac{1}{-e^{\infty}} = \frac{1}{-\infty} = 0 \text{ Convergent}$$