Mohammed Kaabar — Differentiation Techniques (Part I) — Feb. 24, 2016

* Implicit Differentiation

Ex 1| Describe the following equations:

$\boxed{y = e^x}$

$\boxed{y = \sin(x) + x^2 + 10}$

$\boxed{y = x^2 - x + 5}$

Solution: In the above equation, we notice that y is given explicitly in terms of x.

Ex 2| Describe the following equations:

Part a: $\boxed{x^2 + y^2 = 16}$

Part b: $\boxed{x^2 y^3 + \frac{1}{y} - 3 = x}$

Solution: Part a: y is given implicitly in terms of x.

$$y^2 = 16 - x^2 \Longrightarrow y = \pm\sqrt{16 - x^2} \quad \text{So, } y' = \frac{-2x}{\pm 2\sqrt{16 - x^2}}.$$

Part b: y is given implicitly in terms of x.

Ex3 Given $x^2+y^2=16$, then differentiate both sides with respect to x in order to find y'.

Solution: $x^2+y^2=16$

$$2x+2y\cdot y'=0$$

$$\frac{\cancel{2}yy'}{\cancel{2y}}=\frac{-\cancel{2}x}{\cancel{2}y} \Longrightarrow \boxed{y'=\frac{-x}{y}}$$

Notes

$(y^2)'=2(y)^1\cdot y'$
$=2y\cdot y'$

$(y^3)'=3y^2 y' \to \frac{dy}{dx}$

$(x^3)'=3x^2\cdot 1 \to \frac{dx}{dx}$

∎

Ex4 Find y' by differentiating both sides with respect to x for: $y^3+xy=5x$.

Solution: $y^3+xy=5x$ (product rule)

$$3y^2\cdot y'+(xy'+1\cdot y)=5$$

$$3y^2\cdot y'+xy'=5-y$$

$$y'(3y^2+x)=(5-y)$$

So, $\boxed{y'=\frac{(5-y)}{(3y^2+x)}}$

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* Derivatives of Trig. Functions : (look at Handout # 7)

Ex5 Find y' for $y = \sin(\tan x)$.

Solution: $\boxed{y' = \cos(\tan x) \cdot \sec^2 x}$

Ex6 Find y' for $g(x) = \sqrt{\sin^2 x + 1}$.

Solution: $g(x) = (\sin^2 x + 1)^{1/2}$

$$g'(x) = \frac{1}{\cancel{2}}(\sin^2 x + 1)^{-1/2} \cdot (\cancel{2}\sin x \cos x)$$

$$\boxed{g'(x) = \frac{\sin x \cos x}{\sqrt{\sin^2 x + 1}}}$$

Ex7 Find y' for $y = x^2 \sec^4(3x)$. Product Rule

Solution: $y' = 2x(\sec^4(3x)) + x^2(4\sec^3(3x) \cdot \sec(3x)\tan(3x) \cdot 3)$

$$\boxed{y' = 2x\sec^4(3x) + 12x^2(\sec^4(3x)\tan(3x))}$$

Ex8 Find y' for $y = \ln(\boxed{\tan x})$.

Solution: $y' = \frac{\boxed{}'}{\boxed{}} = \boxed{\frac{\sec^2 x}{\tan x}}$