Handout 11

MATH 172 Lab: Sections 7 and 8

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Note: *This handout is a review for exam 3 in MATH 172.*

The following is ***a summary of convergence and divergence tests for series***

Test	*Series*	*Convergent*	*Divergent*	*Notes*
$p-series$	$\sum_{k=1}^{\infty}\frac{1}{k^p}$	$p>1$	$p\leq 1$	You can use to compare with original series as we do in the comparison test
$Divergent\ Test$ $or\ k^{th}-term$	$\sum_{k=1}^{\infty}a_k$	Cannot be used for showing convergence	$\lim_{k\to\infty}a_k\neq 0$	WARNING: We say no-conclusion if $\lim_{k\to\infty}a_k=0$
$Geometric\ Series$	$\sum_{k=1}^{\infty}ar^k$	$\lvert r\rvert<1$	$\lvert r\rvert\geq 1$	If it is convergent, then you need to write the sum as: $S=\frac{a}{1-r}$. You can also use this test for direct and limit comparison tests
$\sum_{k=1}^{\infty}\lvert a_k\rvert$	$\sum_{k=1}^{\infty}a_k$	If $\sum_{k=1}^{\infty}\lvert a_k\rvert$ is convergent, then $\sum_{k=1}^{\infty}a_k$ is also convergent	Cannot be used for showing divergence	If you have a series that has a combination of positive and negative terms, then this test can work well

Telescoping Series	$\sum_{k=1}^{\infty}(c_k - c_{k+1})$	$\lim_{k\to\infty} c_k = L$	Cannot be used for showing divergence	If it is convergent, then you need to write the sum as: $S = c_1 - L$ where c_1 is the initial term (1st term in series)
Alternating Series	$\sum_{k=1}^{\infty}(-1)^{k-1} a_k$	Three Conditions*: a. Alternating b. Decreasing $(0 < a_{k+1} \leq a_k)$. c. $\lim_{k\to\infty} a_k = 0$	Cannot be used for showing divergence	The remainder can be found as follows: $\lvert R_K\rvert \leq a_{K+1}$
Integral Test	$\sum_{k=1}^{\infty} a_k$ $a_k = g(k) \geq 0$ where g is continuous, positive, and decreasing	$\sum_{k=1}^{\infty} a_k$ is convergent if and only if $\int_1^{\infty} g(x)dx$ is convergent	$\sum_{k=1}^{\infty} a_k$ is divergent if and only if $\int_1^{\infty} g(x)dx$ is divergent	The remainder can be found as follows: $0 < R_K < \int_K^{\infty} g(x)dx$
Direct Comparison	$\sum_{k=1}^{\infty} a_k$	If $\sum_{k=1}^{\infty} b_k$ is convergent and $0 \leq a_k \leq b_k$ for every k, then $\sum_{k=1}^{\infty} a_k$ is convergent	If $\sum_{k=1}^{\infty} b_k$ is divergent and $0 \leq b_k \leq a_k$ for every k, then $\sum_{k=1}^{\infty} a_k$ is divergent	$a_k > 0$ and $b_k > 0$
Limit Comparison	$\sum_{k=1}^{\infty} a_k$	$\sum_{k=1}^{\infty} b_k$ is convergent if $\lim_{k\to\infty} \frac{a_k}{b_k} = L > 0$	$\sum_{k=1}^{\infty} b_k$ is divergent if $\lim_{k\to\infty} \frac{a_k}{b_k} = L > 0$	$a_k > 0$ and $b_k > 0$
Ratio Test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \left\lvert\frac{a_{k+1}}{a_k}\right\rvert = L < 1$	$\lim_{k\to\infty} \left\lvert\frac{a_{k+1}}{a_k}\right\rvert = L > 1$	Test is inconclusive if $\lim_{k\to\infty} \left\lvert\frac{a_{k+1}}{a_k}\right\rvert = L = 1$
Root Test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert} = L < 1$	$\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert} = L > 1$	Test is inconclusive if $\lim_{k\to\infty} \lvert a_k\rvert = L = 1$

* To determine whether the alternating series is absolutely convergent or conditionally convergent, you need to use the following Method:

Mohammed Kaabar Binary Method for Alternating Series Test:

$\sum_{k=1}^{\infty} a_k$	$\sum_{k=1}^{\infty} \lvert a_k \rvert$	Type
1 Convergent ←	**1 Convergent**	**Absolutely Convergent**
1 Convergent	**0 Divergent**	**Conditionally Convergent**
0 Divergent →	**0 Divergent**	**Divergent**
0 Divergent	**1 Convergent**	**Inconclusive**