Mohammed Kaabar    Integers as Exponents    Oct 31, 2016
Week 9+10

*General Examples:

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$2^0 = 1$

$2^1 = 2$

* Rule 1: Given that $x$ is a real number and $n$ is a natural number, then we have:

$$\boxed{x^n = x \cdot x \cdot x \cdot x \cdot \ldots \cdot x}$$

* Rule 2: Given that $x$ is a non-zero number, then we have $\boxed{x^0 = 1}$.

* Rule 3: Given that $n$ is a natural number, then we obtain $\boxed{x^{-n} = \frac{1}{x^n}}$

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*How to simplify $(a+c)^2$ ?

Solution: $(a+c)^2 = (a+c)\cdot(a+c)$

$= a\cdot a + a\cdot c + a\cdot c + c\cdot c$

$= \boxed{a^2 + 2ac + c^2}$

Ex1) Simplify: $(2x-1)^2$ ?!

Solution: $(2x-1)^2 = (2x-1)\cdot(2x-1)$

$= (2x)(2x) - (2x)(1) - (1)(2x) + 1$

$= 4x^2 - 2x - 2x + 1$

$= \boxed{4x^2 - 4x + 1}$

Ex2) Simplify: $(x^2+1)^2$ ?!

Solution: $(x^2+1)^2 = (x^2+1)\cdot(x^2+1)$

$= (x^2)(x^2) + x^2 + x^2 + 1$

$= \boxed{x^4 + 2x^2 + 1}$

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*** Rule 4:** If there is a negative number as a base with odd power, then the result stays negative, for example, $(-2)^5 = \boxed{-32}$.

Odd numbers such as 1, 3, 5, 7, 11, 13, 15, 17, 21, ...

*** Rule 5:** If there is a negative number as a base with even power, then the result switches from negative to positive, for example, $(-2)^4 = \boxed{16}$.

Even numbers such as 2, 4, 6, 8, 10, 12, 14, ...

*** Properties of Exponents (Powers):** Assume $x$ and $y$ are non-zero real #s, and $n$ and $m$ are integers

1. Power of Power

$$\boxed{(x^n)^m = x^{nm}}$$

2. Power of Product

$$\boxed{(xy)^n = x^n \cdot y^n \quad \text{or} \quad (xy)^m = x^m \cdot y^m}$$

3. Power of Quotient

$$\boxed{\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}} \quad \text{or} \quad \boxed{\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}}$$

4. Product of Powers

$$\boxed{(x)^n \cdot (x)^m = x^{n+m}}$$

5. Quotient of Powers

$$\boxed{\frac{x^n}{x^m} = x^{n-m}}$$

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* Rule 6: Given that $x$ and $y$ are non-zero real numbers, and $n$ is a natural number, then we have:-

$$\left(\frac{x}{y}\right)^{-n} = \frac{(1)}{\left(\frac{x}{y}\right)^{n}} = (1)\cdot\left(\frac{y}{x}\right)^{n} = \left(\frac{y}{x}\right)^{n}$$

$\boxed{\text{Ex3}}$ Simplify: $7\boxed{y^2}\boxed{x^3}\left(-3\boxed{x^{-3}}\boxed{y^{-5}}\right)$

Solution:

$$= 7(-3)\left(x^{3+(-3)}\right)\left(y^{2+(-5)}\right)$$

$$= -21\cdot\left(x^{0}\right)\cdot\left(y^{-3}\right)$$

$$= -21\cdot(1)\cdot\left(y^{-3}\right)$$

$$= \frac{-21}{y^3} = \boxed{-\frac{21}{y^3}}$$

$\boxed{\text{Ex4}}$ Simplify: $\left(\frac{-x^3}{3\alpha^2 x^7}\right)^3$

Solution:

$$= \frac{\left(-x^3\right)^3}{\left(3\alpha^2 x^7\right)^3} = \frac{-x^9}{27\,\alpha^{2(3)}\,x^{7(3)}}$$

$$= \frac{-x^9}{27\alpha^6 x^{21}} = \frac{-1}{27}\left(\frac{1}{\alpha^6 x^{9-21}}\right)$$

$$= \frac{-1}{27}\left(\frac{1}{\alpha^6 x^{-12}}\right) = \boxed{\frac{-1}{27}\left(\frac{x^{12}}{\alpha^6}\right)}$$

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Ex 5) Simplify: $\left(\frac{2\psi^{2}\beta^{-3}}{\psi^{-3}\beta^{5}}\right)^{-3}$

Solution:

$$\left(\frac{2\psi^{2}\beta^{-3}}{\psi^{-3}\beta^{5}}\right)^{-3} = \left(2\psi^{2-(-3)}\beta^{-3-(5)}\right)^{-3}$$

$$= \left(2\psi^{5}\beta^{-8}\right)^{-3}$$

$$= (2)^{-3}\cdot(\psi^{5})^{-3}\cdot(\beta^{-8})^{-3}$$

$$= (2)^{-3}\cdot(\psi)^{-15}\cdot(\beta)^{24}$$

$$= \frac{\beta^{24}}{(2)^{3}\psi^{15}}$$

$$= \boxed{\frac{\beta^{24}}{8\psi^{15}}}$$

Ex 6) Solve for $\alpha$ to make the following statements true:

Part a: $3^{\alpha}\cdot 3^{3} = 81$

Part b: $\frac{2^{-3}}{2^{\alpha}} = 16$

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Solution:

Part a: $3^{\alpha} \cdot 3^{3} = 81$

$\Rightarrow 3^{\alpha+3} = 81 = 3^{4}$

$\Rightarrow 3^{\alpha+3} = 3^{4}$

Since both sides have the same base which is 3, then $\alpha+3=4$

$\alpha = 4-3$

$\boxed{\alpha = 1}$

Check: $3^{1} \cdot 3^{3} = 3^{1+3} = 3^{4} = \boxed{81}$ ✓

Part b: Solution:

$\frac{2^{-3}}{2^{\alpha}} = 16 \Rightarrow 2^{-3-\alpha} = 16 = 2^{4}$

$\Rightarrow 2^{-3-\alpha} = 2^{4}$

$\Rightarrow$ Since both sides have the same base which is 2, then the powers (exponents) are equivalent as follows: $-3-\alpha = 4$

$-3-4 = \alpha$

$\boxed{-7 = \alpha}$

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<u>Check:</u> $\frac{2^{-3}}{2^{-7}} = 2^{-3-(-7)} = 2^{-3+7} = 2^{4} = \boxed{16}$ ✓

## * Scientific Notation

<u>Definition:</u> It's an expression written as:

$M \times 10^{n}$ where $n$ is an integer, $M$ is greater than or equal 1 and less than 10

$\Rightarrow 1 \le M < 10$.

**Ex7** Convert the following to scientific notation:

Part a: 22,000

Part b: 0.00023

<u>Solution:</u>

a. $22{,}000 = \boxed{2.2 \times 10^{4}}$

b. $0.00023 = \boxed{2.3 \times 10^{-4}}$