Final Exam Study Guide

MATH 172 Lab: Sections 7 and 8

Lab Instructor (TA): Mohammed Kaabar

Student's Name:----------

Student's ID:----------

Note: *This study guide contains comprehensive practice questions for the final exam in Calculus II.*

Question 1: Find the volumes of the region generated by revolving $y = \sqrt{x}$, $y = 2$, and $y - axis$ about the following:

Part a: $x - axis$

Part b: $y - axis$

Part c: $y = -1$

Question 2: Evaluate the following indefinite integrals:

Part a: $\int x^{-3}\ln(x)\,dx$.

Part b: $\int \sin^5(x)\cos^3(x)\,dx$.

Part c: $\int \frac{dx}{\sqrt{x^2-4}}$

Part d: $\int \tan^{-1}\left(\sqrt{x}\right) dx$.

Part e: $\int \frac{x+2}{x^3+x} dx$.

Question 3: Evaluate the following definite integral:

$$\int_{3}^{4} \frac{dx}{x^2 - 6x + 10}$$

Question 4: Write out the partial fraction decomposition of the following expression. **DO NOT SOLVE FOR COEFFECIENTS**:

$$\frac{x^3 + 2x^2 - x + 1}{x(x^2 - 4)(x^2 + 6)^2}$$

Question 5: Evaluate the following improper integrals and determine whether the integral converges or diverges:

Part a:

$$\int_{1}^{\infty} \frac{e^{-x^3}}{x^{-2}}\,dx$$

Part b:

$$\int_{-1}^{8} x^{-3}\,dx$$

Question 6:

Part a: Consider the following sequence $\left\{\frac{k^2-1}{k+k^2}\right\}_{k=2}^{\infty}$. Show that the sequence is increasing and determine whether the sequence is bounded above, and bounded below.

Part b: Consider the following sequence $\left\{\frac{3^k}{(2k-1)!}\right\}_{k=1}^{\infty}$. Determine whether the sequence is increasing or decreasing or neither.

Question 7: Determine if the series diverges or converges. Be sure to explain which test you use:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

Question 8: How many terms of the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$$

are needed to approximate the sum of the series with maximum error of $(0.5)10^{-4}$?

Question 9: Determine if the following series diverges or converges. Be sure to explain which test you use:

Part a:

$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$$

Part b:

$$\sum_{n=1}^{\infty} \frac{n+2}{\left(n^{\frac{3}{2}}+n+1\right)(n+1)}$$

Part c:

$$\sum_{n=2}^{\infty} \frac{\ln(n)}{\left(n^{\frac{3}{2}}+1\right)}$$

Question 10: Determine the radius and interval of convergence for the following series:

$$\sum_{m=0}^{\infty} \frac{m^2}{m!}(x+1)^m$$

Question 11: Find the following using only Taylor's series:

$$\lim_{x \to 0} \left(\frac{\sin(x^3) - x^3}{x^9} \right)$$

Question 12:

Part a: Change to rectangular coordinates and insert a graph below your result: $r = 6\cos(\theta)$.

Part b: Sketch a graph for $r = 2 + 4\cos(\theta)$ using symmetry tests.

Part c: Find the area inside the inner loop of the polar coordinates: $r = 1 + 2\cos(\theta)$.

Question 13: Find the parametric equations for the line segment joining the points $(1,2)$ and $(4,7)$.

Question 14:

Part a: Given $\vec{m} = \langle 2,4 \rangle$ and $\vec{w} = \langle 3,1 \rangle$. Find the following:

1- $\vec{m} + \vec{w}$

2- $\vec{m} - 2\vec{w}$

3- $\|5\vec{w} - 2\vec{m}\|$

Part b: Given $\vec{v} = \langle 6,8 \rangle$. Find a unit vector $\vec{u}$ in the same direction of $\vec{v}$.

Part c: Given $\vec{m} = \langle 3,2,0 \rangle$ and $\vec{w} = \langle -2,4,3 \rangle$. Find the following:

1- $\vec{m} \cdot \vec{w}$

2- The angle θ between $\vec{m}$ and $\vec{w}$.

3- $\vec{m} \otimes \vec{w}$

Part d: Find the distance from the point $A(2,0,1)$ to the line joining $B(1,-2,2)$ and $C(3,0,2)$.

Good Luck in the Final Exam

Best of Luck

Mohammed K A Kaabar