Week 1
(Part 1)

Trigonometric Functions

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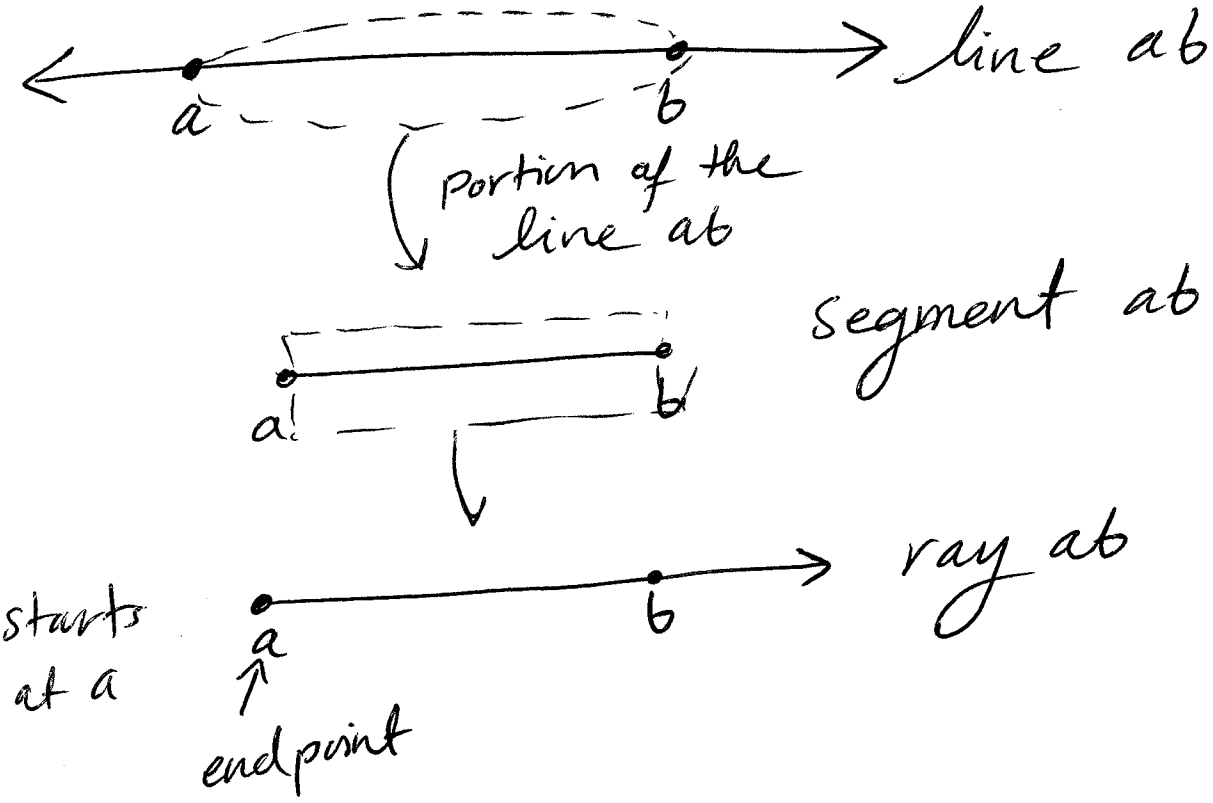
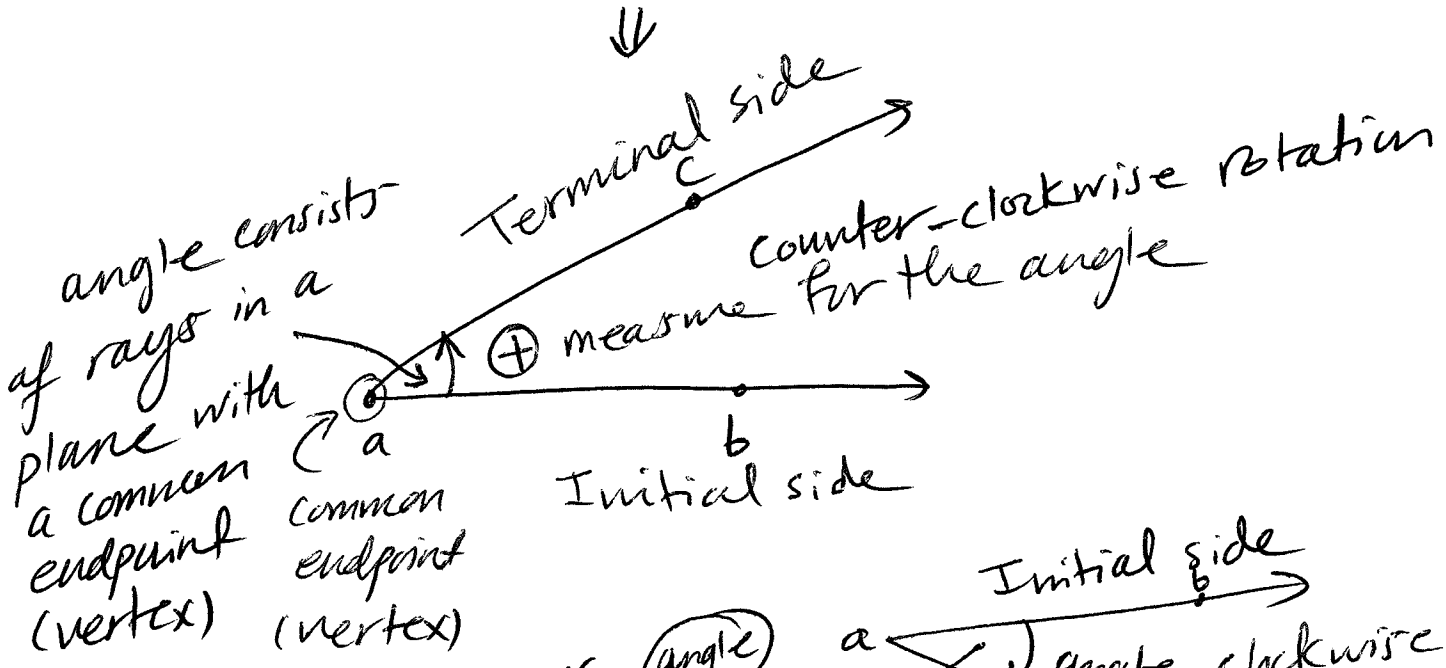
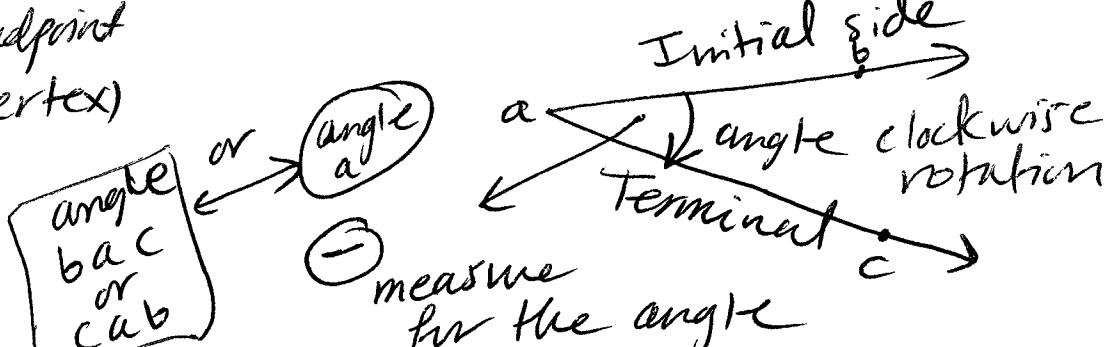
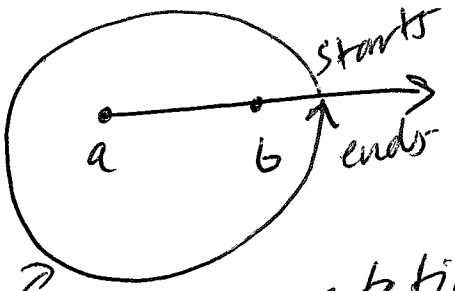
* Angles







* there are two common units in measuring angles: Degree and Radian



ab ray

complete rotation of ray ab $\Rightarrow$ 360 degrees

one degree $\Leftrightarrow$ we write it as 1°

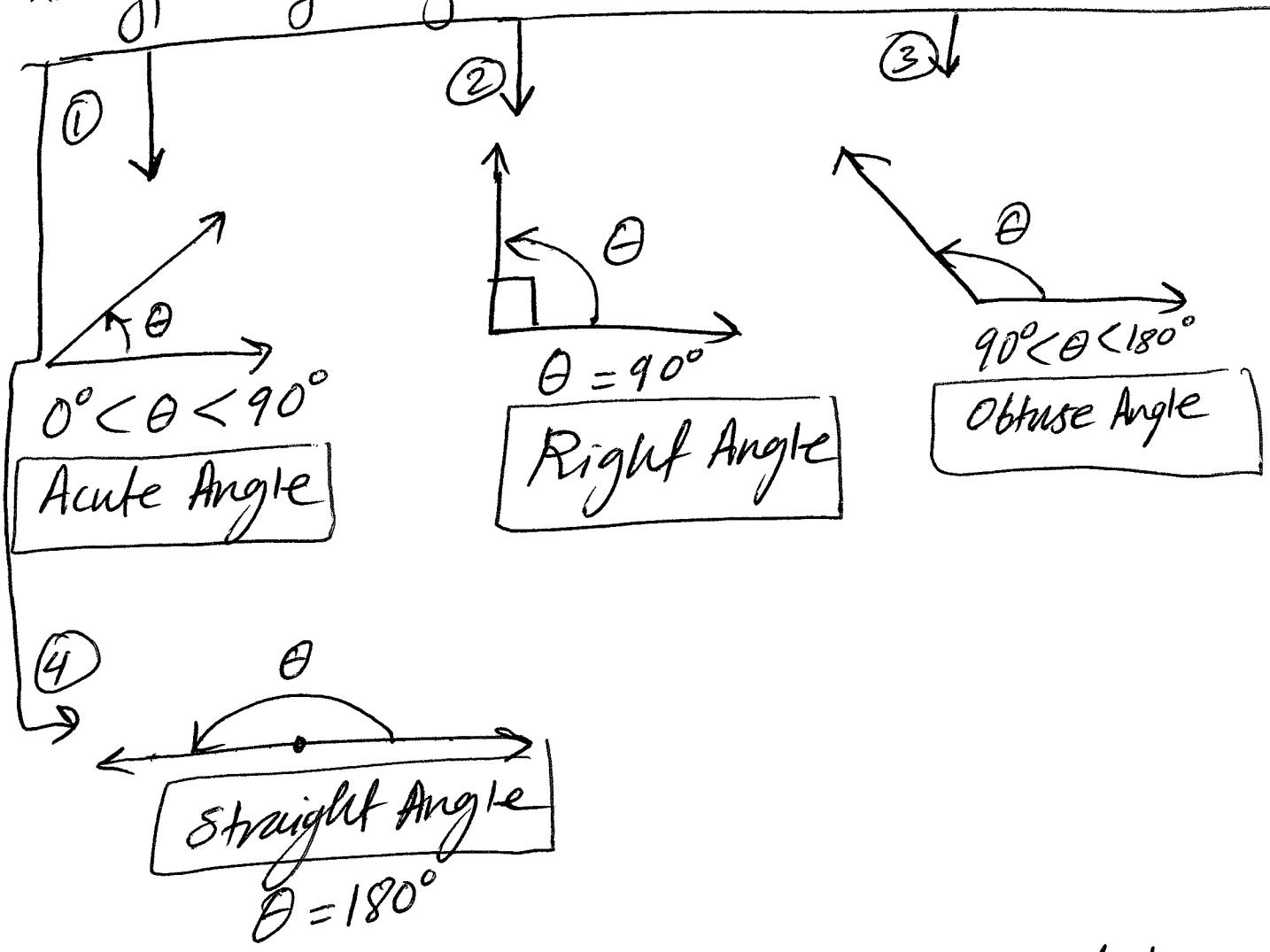
1° represents $\frac{1}{360}$ of complete rotation

90 degree $\Leftrightarrow$ 90° $\Leftrightarrow$ $\frac{90}{360} = \frac{1}{4}$ of complete rotation

180 degree $\Leftrightarrow$ 180° $\Leftrightarrow$ $\frac{180}{360} = \frac{1}{2}$ of complete rotation

* It's very common to name angles using Greek letter θ (theta).

* Types of Angles



* Note:- If the sum of the measures of two positive angles is $\boxed{90^\circ}$ then these angles are complements (complementary) of each other.

* On the other hand, if the sum of the measures of two positive angles is $\boxed{180^\circ}$ then these angles are supplements (supplementary) of each other.

Ex1 Find the complement and supplement of an an angle measuring 60°.

Solution:-

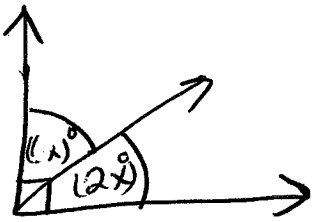
① Complement:

$$90^\circ - 60^\circ = 30^\circ$$

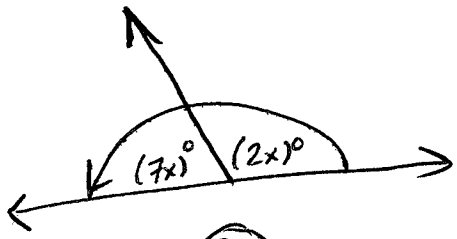
② Supplement:

$$180^\circ - 60^\circ = 120^\circ$$

Ex2 Given the following



①



②

Find the measure of each marked angle in ① and ②.

Solution:-

① $7x + 2x = 90$ (supplementary)

$$\frac{9x}{9} = \frac{90}{9}$$

$$\boxed{x = 10}$$

$2(10) = 20^\circ$

$7(10) = 70^\circ$

total $\Rightarrow 20^\circ + 70^\circ = 90^\circ$ right angle

② $7x + 2x = 180$ (complementary)

$9x = 180 \Rightarrow \boxed{x = 20}$

$7(20) = 140^\circ$

$2(20) = 40^\circ$

total $\Rightarrow 140^\circ + 40^\circ = 180^\circ$ straight angle

Note: $1^\circ = \frac{1}{360}$ of a complete rotation

$\Rightarrow$ 1 minute $\Leftrightarrow 1' = \frac{1}{60}$ of a degree

$\Rightarrow$ 1 second $\Leftrightarrow 1'' = \frac{1^\circ}{60}$ of a minute or $\boxed{60' = 1^\circ}$

So, $1'' = \frac{1'}{60} = \frac{1^\circ}{60 \times 60} = \frac{1^\circ}{3600}$

which is $\boxed{60'' = 1'}$

Ex3] Calculate the following

(a) $22^\circ 12' + 15^\circ 10'$

(b) $90^\circ - 70^\circ 12'$

Solution:-

(a) $22^\circ 12'$

$+ 15^\circ 10'$

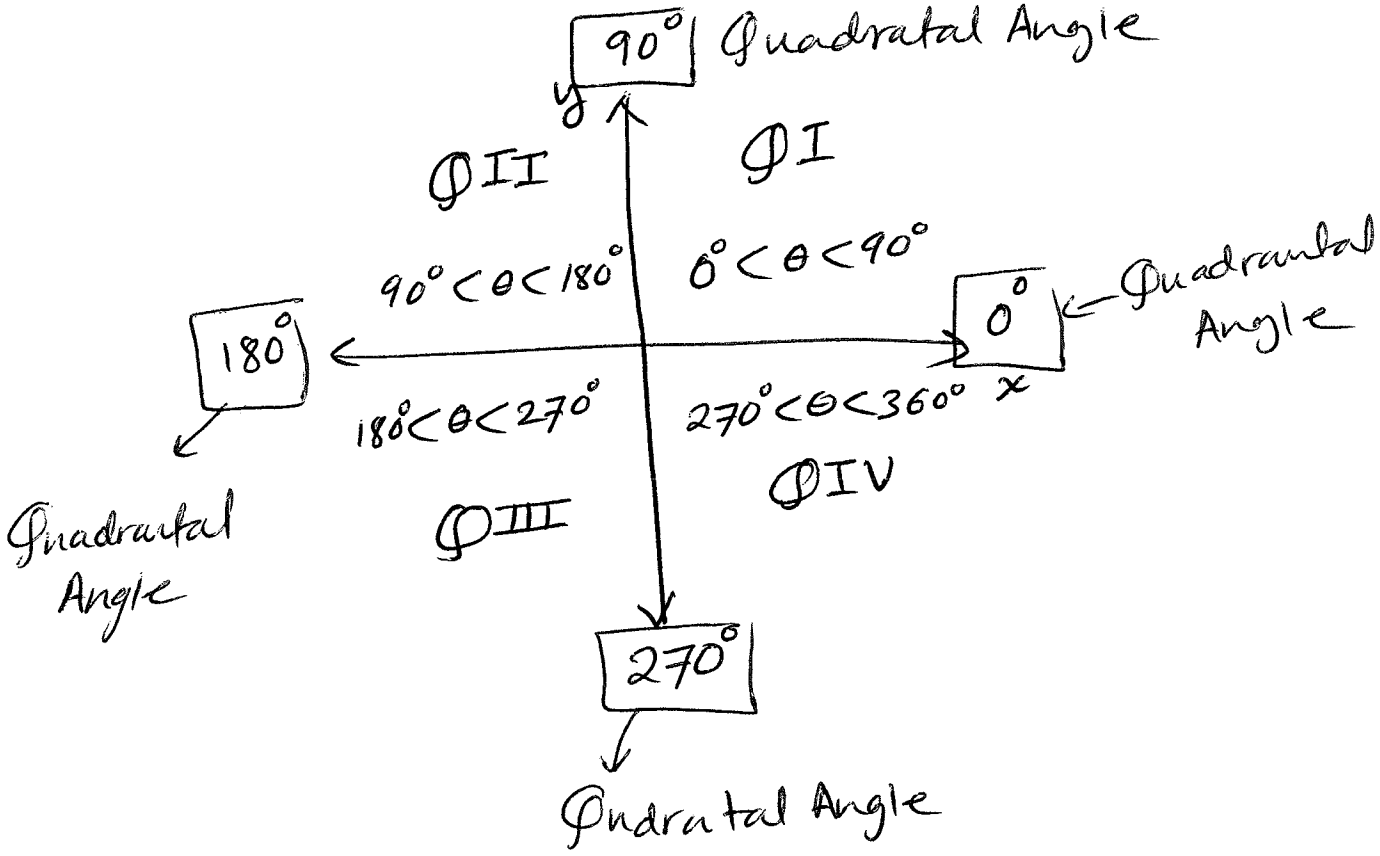
$\boxed{37^\circ 22'}$

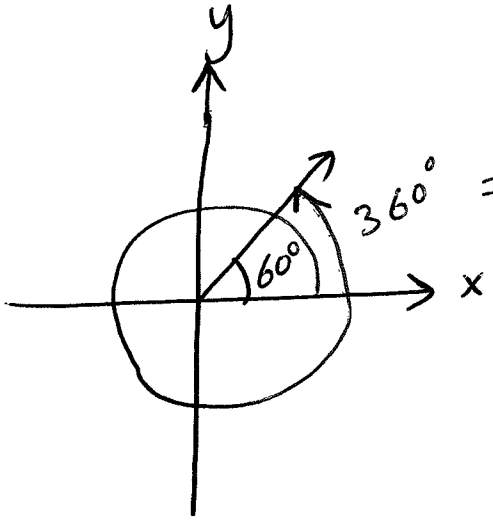
(b) $90^\circ = 89^\circ + 1^\circ = 89^\circ + 60' = \boxed{89^\circ 60'}$

So, $89^\circ 60'$

$- 70^\circ 12'$

$\boxed{19^\circ 48'}$





$60^\circ + 360^\circ = 420^\circ$

same initial side
terminal side
$\oplus$ Larger than 360°

differes in rotations

Coterminal Angles

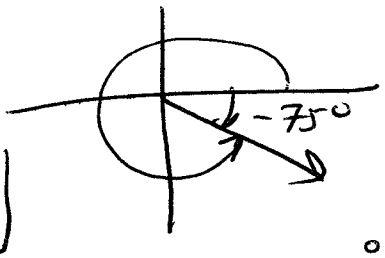
Ex 4] Find the angles of least positive measure that are coterminal with each angle:

(a) 908° (b) -75° (c) -800°

Solution:-

(a) $908^\circ - \underbrace{(2)(360^\circ)}_{720^\circ} = \boxed{188^\circ}$ $\quad 0^\circ < \theta < 360^\circ$

(b) $360^\circ + (-75^\circ) = \boxed{285^\circ}$ $\quad 0^\circ < \theta < 360^\circ$



(c) $(-800) + (3)(360) = \boxed{280^\circ}$

$0^\circ < \theta < 360^\circ$