Implementation Notes

Open with a conceptual review, much like what is paraphrased in the beginning of the lab. Go through the first example on the board either as a class or by yourself. Now pass out the lab, get students into groups of no more then four or possibly five and circulate the room while they work. As you circulate and monitor student progress choose groups to present solutions that you feel are particularly useful for instructional purposes. Remember its always a good idea to have students see other students work and interpret logic for themselves.

Practice with Volumes

Research shows that students **learn** when they **MAKE MISTAKES** and not when they get it right. SO DON'T BE AFRAID TO MAKE MISTAKES, because MISTAKES ARE GOOD!

Volumes

Conceptually the main idea behind these sections is to think of the volume of a solid as the infinite addition of the areas of its cross sections much like the idea behind Riemann sums. The trick then is to write the area of an arbitrary cross section as a integrable function. This idea inspires the formal definition below:

- Def: Let V be the volume of a solid that lies between $x = a$ and $x = b$. If the cross-sectional area $A(x)$ of the solid through x and perpendicular to the $x-$axis is an integrable function then the volume of the solid can be expressed as:

$$V = \int_a^b A(x)dx$$

Your job in these sections is to find $A(x)$ or $A(y)$ (depending on whether your integrating with respect to x or y respectively), and then integrate it. In order to do this you will need to recall a couple of formulas.

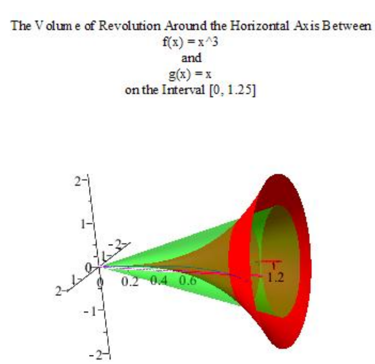
- Area of a Rectangle with base b and height h — bh
- Area of a Triangle with base b and height h — $\frac{1}{2}bh$
- Area of a Circle with radius r — πr^2
- Area of a Washer or Donut (Circle minus Circle) with outer radius r_o and inner radius r_i — $\pi\left(r_o^2 - r_i 2\right)$
- Area of a the surface of a cylinder with radius r and height h — $2\pi rh$

If you can remember these formulas and always draw a picture then you will never have a problem you can't handle.

Example:

Find the volume of the solid obtained by rotating the region bounded by $f(x) = x^3, g(x) = x, x \geq 0$ about the x-axis.

Solution: First we sketch a graph of the volume.

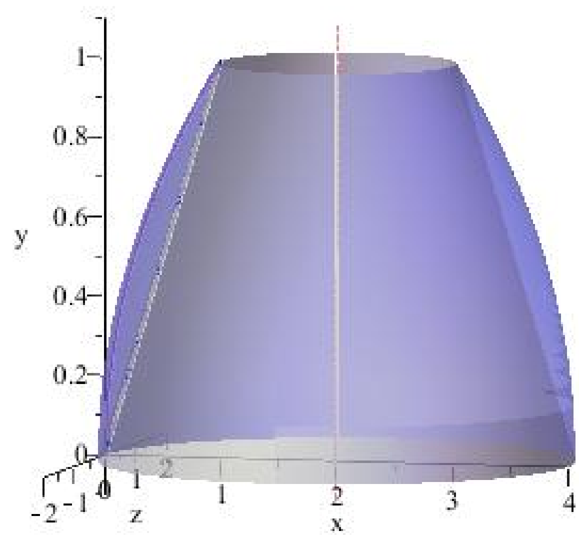


Clearly $f(x) = g(x)$ when $x = 0$ and $x = 1$, and if we take any cross section perpendicular to the x-axis in that range we will get a washer with outer radius $r_o = x$ and inner radius $r_i = x^3$. The area of the cross-section $A(x)$ is therefore $A(x) = \pi\left(r_o^2 - r_i^2\right) = \pi\left(x^2 - x^6\right)$. Since $A(x)$ is integrable we can find the volume by integrating $A(x)$ from 0 to 1 as follows:

$$V = \int_0^1 A(x)dx = \int_0^1 \pi\left(x^2 - x^6\right)dx = \left[\pi\left(\frac{x^3}{3} - \frac{x^7}{7}\right)\right]_0^1 = \frac{\pi}{3} - \frac{\pi}{7}$$

In Lab Exercises

A.) Using the given method, find the volume of the solid obtained by rotating the region bounded by $y = x, y = \sqrt{x}$ about the line $x = 2$ as shown in the figure below. Verify both methods give the same result.



i. Slicing method

ii. Shell method

B.) Find the volume of the solid whose base is enclosed by the circle $x^2 + y^2 = 1$ and whose cross sections taken perpendicular to the base are

i.) Semicircles

ii.) Squares

iii.) Equilateral Triangles

C.) Using either slicing method or shell method, verify that the volume of a cone whose base is a circle of radius r and whose height is h is given by $\frac{h}{3}\pi r^2$.