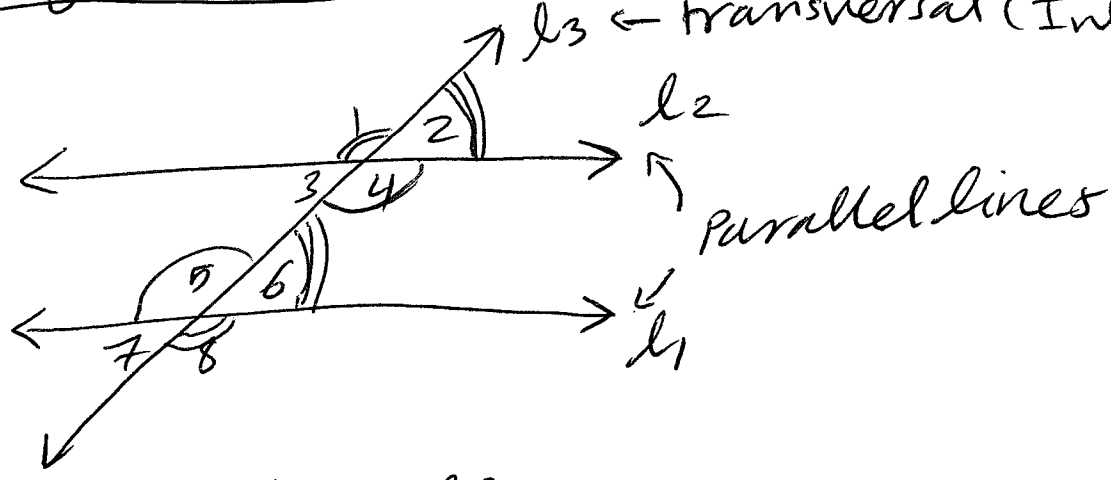
Week 1 (Part 2) — Trigonometric Functions (Part 2) — M. Kaabar

* Angle Relationships and Similar Triangles



Angle Measure

$2 = 6$
$4 = 8$
$1 = 5$
$3 = 7$
} corresponding Angles

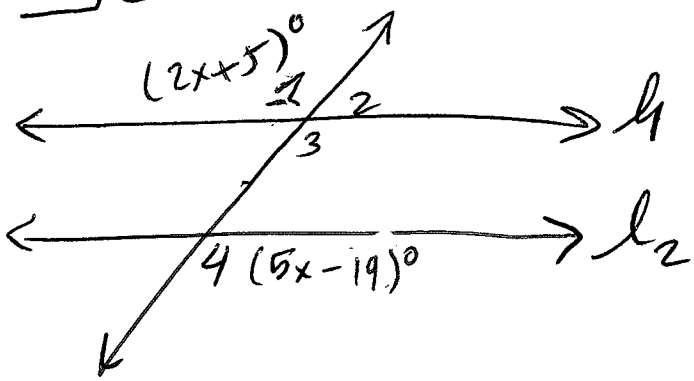
$3 = 6$
$4 = 5$
← Alternate Interior Angles

$1 = 8$
← Alternate Exterior Angles

$2 = 7$

4 and 6
3 and 5
→ add to 180°
Interior Angles on same side of transversal.

Ex 1 Given the following:-



Find the measure of angles 1, 2, 3, and 4.

Solution

$1 = 4 \Rightarrow$ So, $(2x+5) = (5x-19)$

$(5x-2x) = (5+19)$

$3x = 24$

$x = \frac{24}{3} = \boxed{8}$

So, plug $x=8$ in 1, we get: $2(8)+5$

$= 16+5$

$= \boxed{21^\circ}$

" " " " 4, we get: $5(8)-19$

$= 40-19$

$= \boxed{21^\circ}$

For ②, ② is the supplement of 21°, so

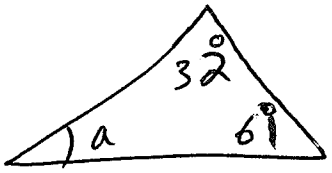
the angle 2 is $180^\circ - 21^\circ = \boxed{159^\circ}$

For ③, $\boxed{1=3}$, so ③ is $\boxed{21^\circ}$

(vertical to each other)

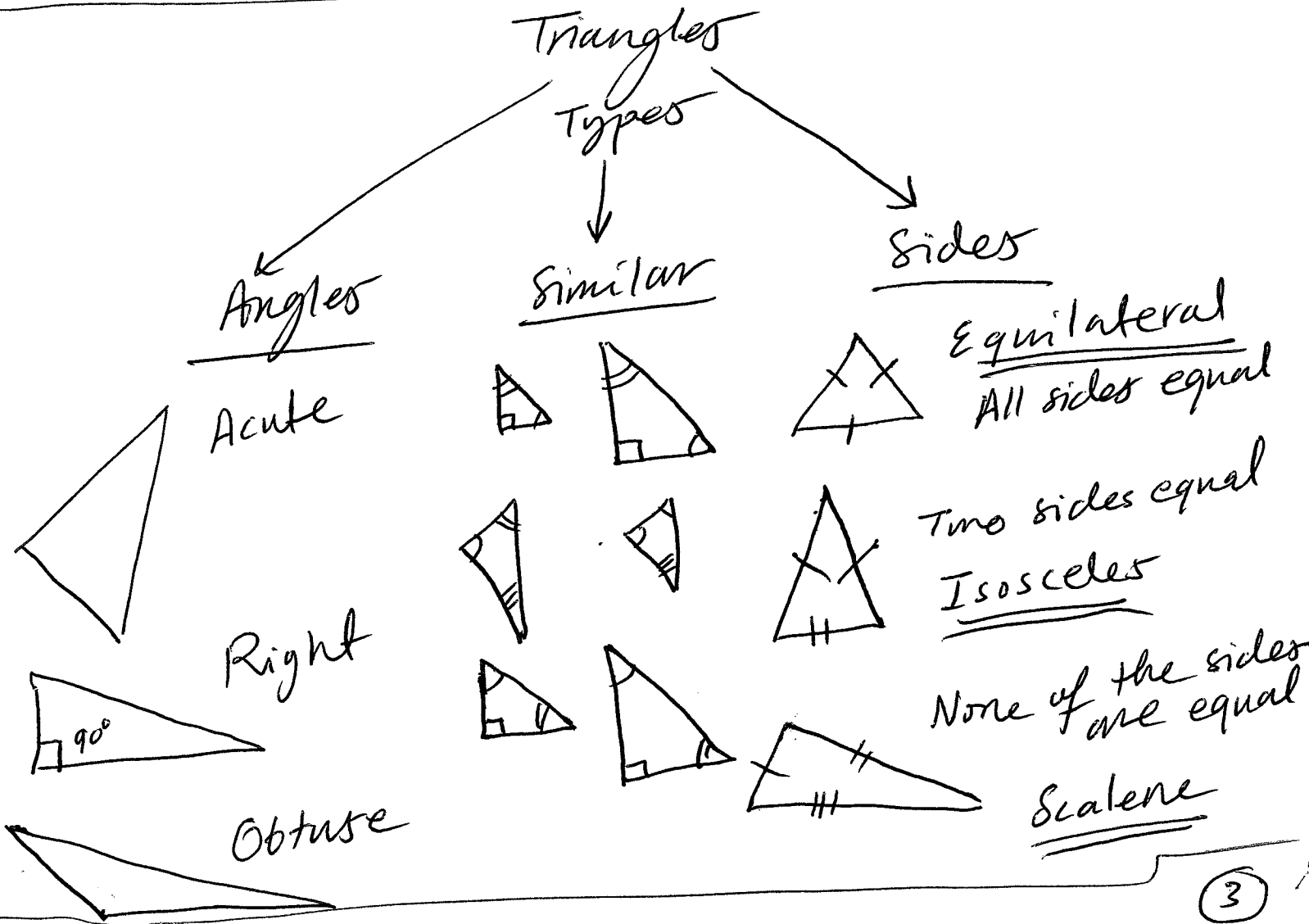
* Note: The sum of the angles' measures of any triangle is always 180°.

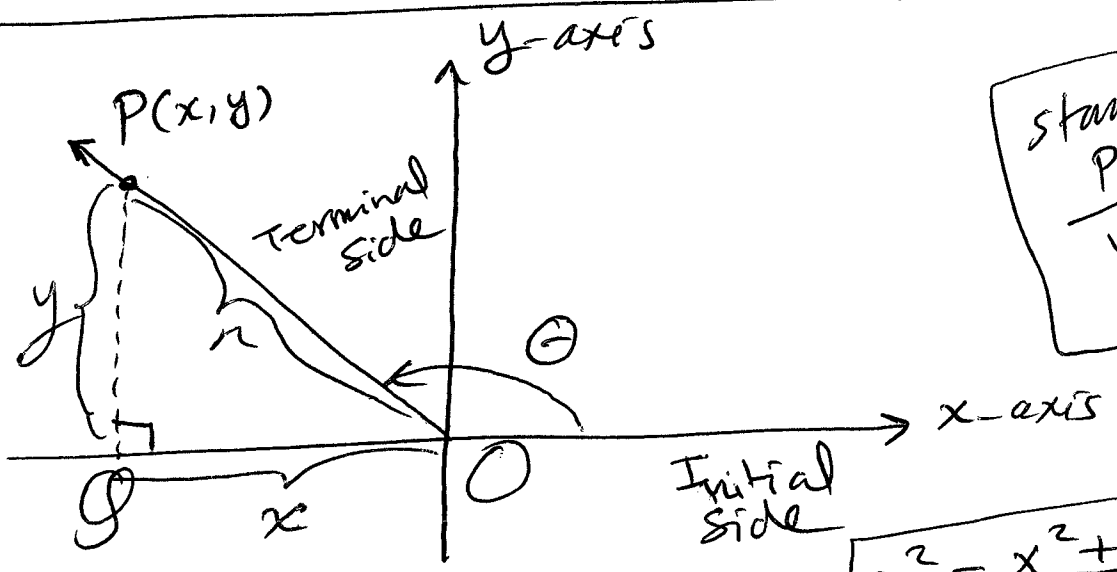
Ex2) Find a for



Solution:

$$a = 180^\circ - (32^\circ + 61^\circ)$$
$$= 180^\circ - (93^\circ)$$
$$= 87^\circ$$



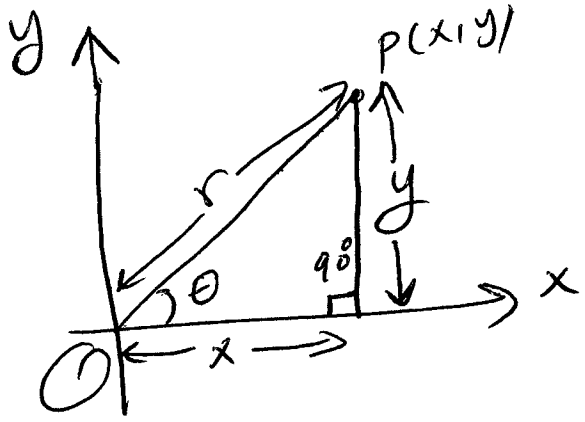


Standard position of θ
vertex at (0,0)
Initial side at ⊕ axis

$r^2 = x^2 + y^2$ Pythagorean Theorem

$\sqrt{r^2} = \sqrt{x^2 + y^2}$

$r = \sqrt{x^2 + y^2}$



$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} \quad \text{where } x \neq 0$$

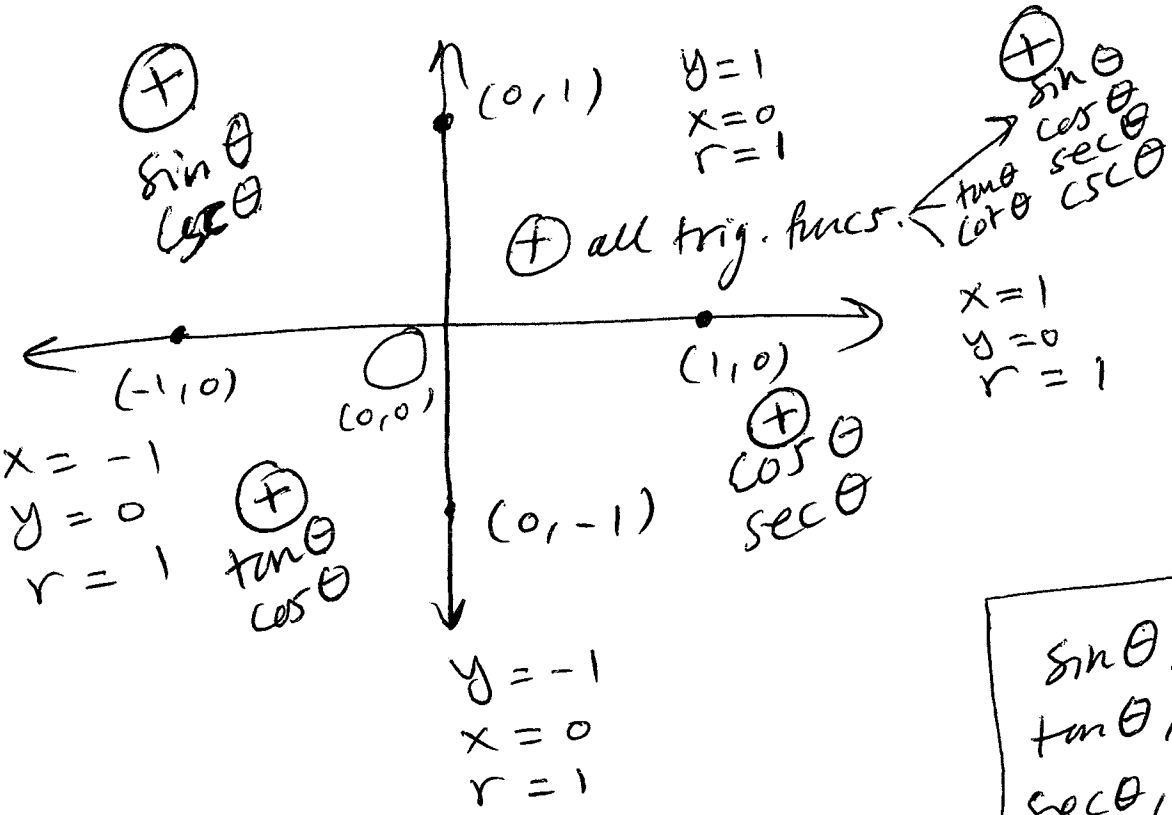
$= \frac{\sin\theta}{\cos\theta}$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{y}{r}} = \frac{r}{y} \quad \text{where } y \neq 0$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{x}{r}} = \frac{r}{x} \quad \text{where } x \neq 0$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{y}{x}} = \frac{x}{y} \quad \text{where } y \neq 0$$

$= \frac{\cos\theta}{\sin\theta}$



$\sin\theta, \cos\theta$ $[-1,1]$

$\tan\theta, \cot\theta$ $(-\infty,\infty)$

$\sec\theta, \csc\theta$ $(-\infty,-1]\cup[1,\infty)$

$$-1 \le \sin\theta \le 1$$

$$-1 \le \cos\theta \le 1$$

$$\sec\theta \le -1 \quad \text{or} \quad \sec\theta \ge 1$$

$$\csc\theta \le -1 \quad \text{or} \quad \csc\theta \ge 1$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cos^2\theta + 1 = \csc^2\theta$$

} Pythagorean Identities