Quiz 6

MATH 172 Lab: Sections 8

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5

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Note: This quiz covers Alternating Series Test and Taylor Polynomials.

Show your work and circle your answers. Neatness and organization count!

Question 1: (3 points) Determine whether the following series diverges, converges conditionally, or converges absolutely:

Thus, it's absolutely convergent.

$$\sum_{\Omega=1}^{\infty}(-1)^{\Omega}\Omega^{2}e^{-\Omega} = \sum_{\Omega=1}^{\infty}\frac{(-1)^{\Omega}\Omega^{2}}{e^{\Omega}}$$

Compare $\sum_{\Omega=1}^{\infty}\frac{(-1)^{\Omega}\Omega^{2}}{e^{\Omega}}$ with $\sum_{\Omega=1}^{\infty}\frac{\Omega^{2}}{e^{\Omega}}$

$$\lim_{\Omega\to\infty}\left|\frac{(\Omega+1)^2}{e^{(\Omega+1)}}\cdot\frac{e^{\Omega}}{\Omega^2}\right| = \frac{(\Omega+1)^2}{\Omega^2 e} = \frac{\cancel{\Omega^2}}{\cancel{\Omega^2}e} = \underbrace{\frac{1}{e}}_{\cong 0.36} < 1 \text{ Convergent}$$

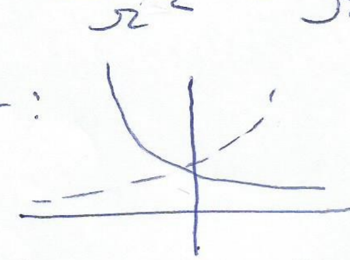
by Ratio Test.

OR Root Test: $\frac{(\sqrt[\Omega]{\Omega})^2}{e} = \frac{1}{e} < 1$ Convergent

OR Direct Comparison Test: $\frac{e^{-\Omega}}{\Omega^2} \le \frac{1}{\Omega^2}$ Convergent

Alternating Series test

OR Graph Test:



Convergent.

So, it's convergent by

(i) Alternating

(ii) $\lim_{\Omega\to\infty}\frac{\Omega^2}{e^{\Omega}} \overset{L'H}{=} \frac{2\Omega}{e^{\Omega}} = \frac{2}{e^{\Omega}} = \frac{2}{\infty} = 0$

(iii) Assume $f(x) = \frac{x^2}{e^x}$

$f'(x) < 0$ decreasing

Question 2: (2 points) Find the 3rd degree Taylor Polynomial for $f(\Sigma) = \tan^{-1}(\Sigma)$ centered at $\Sigma = 0$.

$$f(\Sigma) = \tan^{-1}(\Sigma) \longrightarrow f(0) = \tan^{-1}(0) = 0$$

$$f'(\Sigma) = \frac{1}{1+\Sigma^2} = (1+\Sigma^2)^{-1} \longrightarrow f'(0) = 1$$

$$f''(\Sigma) = -(1+\Sigma^2)^{-2} \cdot 2\Sigma \longrightarrow f''(0) = 0$$

$$f'''(\Sigma) = 2(1+\Sigma^2)^{-3} \cdot 2\Sigma - (1+\Sigma^2)^{-2} \cdot 2 \longrightarrow f'''(0) = -2$$

$$\tan^{-1}(\Sigma) \approx 0 + \frac{1(\Sigma)}{1!} + \cancelto{0}{\frac{0(\Sigma^2)}{2!}} - \frac{2(\Sigma^3)}{3!}$$

Therefore, $\boxed{\tan^{-1}(\Sigma) \approx \Sigma - \frac{\Sigma^3}{3}}$ ☒