

Example about Growth/Decay from a textbook:
(A First Course in Differential Equations) by Zill.

Problem # 6 page: 89

100 mg of radioactive substance. After 6 hours, the mass has decreased by 3%. Rate of decay is proportional to the amount of the substance presented at time t , find the amount remaining after 24 hours.

Solution:

$A(t)$ = Present Amount of the substance at time t .

t = # of hours.

$$\frac{dA}{dt} = \alpha A$$

$$A(0) = 100 \text{ mg}$$
$$A(6) = 100 - \frac{3}{100} \times 100 = 97 \text{ mg}$$

So, $\frac{dA}{dt} = \frac{\alpha}{A^{-1}}$ "Separable Method"

$$\int A^{-1} dA = \int \alpha dt$$

$$\int A^{-1} dA - \int \alpha dt = C$$

$$\ln|A| - \alpha t = C$$

$$\ln|A| = \alpha t + C \Rightarrow A = e^{\alpha t + C} = e^{\alpha t} \cdot \underbrace{e^C}_{\text{Assume } = C} = Ce^{\alpha t} \Rightarrow A = Ce^{\alpha t}$$

To find α and C , we do the following: $A(0) = Ce^{\alpha(0)} = 100$
 $\Rightarrow C = 100$

$$A(6) = 100e^{\alpha(6)} = 97 \Rightarrow e^{6\alpha} = \frac{97}{100} = 0.97 \Rightarrow \ln e^{6\alpha} = \ln(0.97) \Rightarrow 6\alpha = \ln(0.97)$$
$$\Rightarrow \alpha = \frac{\ln(0.97)}{6}$$