Handout 8

MATH 172 Lab: Sections 7 and 8

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Note: This handout covers 6 different convergence tests.

Instruction: Work in groups to solve the following mathematical problems. DON'T AFRAID TO MAKE MISTAKES BECAUSE WE LEARN FROM OUR MISTAKES!

Problem 1: Investigate the sequence: $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$.

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right) = \frac{\sin(1/n)}{1/n} \Rightarrow$$ Suppose $x = 1/n$

then, $\Rightarrow = \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \neq 0$ $\boxed{\text{Divergent}}$

Notes

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

while

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

Problem 2: Investigate the sequence: $\sum_{n=1}^{\infty} (1+n)^{\frac{1}{n}}$.

$$\sum_{n=1}^{\infty} (1+n)^{1/n} \Rightarrow \lim_{n \to \infty} (1+n)^{1/n} = e^{\ln(1+n)^{1/n}}$$

$$= e^{\frac{\ln(1+n)}{n}} \Rightarrow \lim_{n \to \infty} e^{\frac{\ln(1+n)}{n}} = e^{0} = 1 \neq 0$$ $\boxed{\text{Divergent}}$

$$\lim_{n \to \infty} \frac{\ln(1+n)}{n} \overset{L'H}{=} \frac{\frac{1}{1+n}}{1} = 0$$

Notes

$$\lim_{n \to \infty} (1+n)^{1/n} = e$$

and

$$\lim_{n \to 0} \left(1+\frac{1}{n}\right)^{n} = e$$

Definition of e

Problem 3: Investigate the sequence: $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$.

$$\ln(n) \leq n^{1/2}$$

$$\frac{\ln(n)}{n^2} \leq \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$$

p-series $p = \frac{3}{2} > 1$ $\boxed{\text{Convergent}}$

Notes

$\ln(n) \leq n^{\alpha}$ where α is a positive number

Problem 4: Investigate the sequence: $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$.

$$\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}} \Rightarrow \sum_{n=1}^{\infty} \frac{2n^2}{\sqrt{n^5}} = 2\sum_{n=1}^{\infty} \frac{n^2}{n^{5/2}} = 2\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

p-series $p = 1/2 < 1$ $\boxed{\text{Divergent}}$

OR

$$\Rightarrow \lim_{n\to\infty} \frac{(2n^2+3n)}{\sqrt{5+n^5}} \cdot \frac{\sqrt{n^5}}{2n^2} = \lim_{n\to\infty} \frac{\cancel{2n^2}}{\cancel{n^{5/2}}} \cdot \frac{\cancel{\sqrt{n^5}}}{\cancel{2n^2}}$$

$= 1 > 0$ $\boxed{\text{Divergent}}$.

Problem 5: Investigate the ~~sequence~~: $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$.

$(2n+2)! = (2(n+1))! = 2(n+1)!$

or

$\boxed{(2n+2)! = (2n+2)(2n+1)\cdot\cancel{(2n)!}}$

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!} = \lim_{n\to\infty} \frac{(2n+2)!}{(n+1)!(n+1)!} \cdot \frac{\cancel{n!}\,\cancel{n!}}{\cancel{(2n)!}}$$

$(n+1)n!$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(n+1)(n+1)} = \lim_{n\to\infty} \frac{4n^2+2n+4n+2}{n^2+n+n+1} = \frac{4\cancel{n^2}}{\cancel{n^2}}$$

$= 4 > 1$ $\boxed{\text{divergent}}$

Problem 6: Investigate the sequence: $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

$$\sum_{n=1}^{\infty} \frac{n^n}{n!} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{\cancel{(n+1)!}} \cdot \frac{\cancel{n!}}{n^n} = \frac{(n+1)^n \cancel{(n+1)}}{n^n \cancel{(n+1)}} = \left(\frac{n+1}{n}\right)^n$$

$(n+1)$

$$\Rightarrow \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = \lim_{n\to\infty} e^{\ln(1+1/n)^n}$$

$$e^{n\ln(1+1/n)} \Rightarrow \lim_{n\to\infty} e^{\frac{\ln(1+1/n)}{1/n}} = \lim_{n\to\infty} e^1 = e > 1$$

$\boxed{\text{Divergent}}$

L'H $\quad \dfrac{\dfrac{-1/n^2}{1+1/n}}{-1/n^2} = 1$

Notes

- $(n+1)^3 = (n+1)^2 \cdot (n+1)^1$
- $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

Problem 7: Investigate the sequence: $\sum_{n=1}^{\infty} \frac{n^n}{3^{(1+3n)}}$.

$$\sum_{n=1}^{\infty} \frac{n^n}{3^{(1+3n)}} = \sum_{n=1}^{\infty} \frac{n^n}{3^1 \cdot 3^{3n}} = \frac{1}{3}\boxed{\sum_{n=1}^{\infty} \frac{n^n}{3^{3n}}}$$

$* \; 3^{(1+3n)} = \boxed{3^1 \cdot 3^{3n}}$ Note

$$\Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{n^n}{3^{3n}}} = \frac{n}{3^3} = \frac{n}{27} = \frac{\infty}{27} = \infty > 1$$

$\boxed{\text{divergent}}$.

Good Luck in "Practice with Series" Lab. Remember: there will be a question in exam 3 from this lab
Best Regards,
Mohammed Kaabar