Ex 1 $\lim_{x\to 1} \frac{x-1}{x^2-1}$. Evaluate it?! $\frac{1-1}{1^2-1} = \frac{0}{0}$ (Undefined)

Solution:

$$\lim_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$$

Ex 2 $\lim_{x\to 2} \frac{x^4-16}{4-2x}$. Evaluate it?! $\frac{16-16}{4-4} = \frac{0}{0}$ Undefined

Solution:

$$\lim_{x\to 2} \frac{x^4-16}{4-2x} = \frac{(x^2-4)(x^2+4)}{2(2-x)} = \lim_{x\to 2} \frac{\overset{-1}{\cancel{(x-2)}}(x+2)(x^2+4)}{2\cancel{(2-x)}} =$$

$$= \lim_{x\to 2} \frac{-(x+2)(x^2+4)}{2} = \frac{-(4)(4+4)}{2} = \frac{-4(8)}{2} = \frac{-32}{2} = -16$$

* Some Important Notes:

① $\frac{0}{0} = \frac{\text{Exactly Zero}}{\text{Exactly Zero}} \overset{\text{implies}}{\Longrightarrow}$ Undefined

$\frac{\tilde{0}}{\tilde{0}} = \underbrace{\frac{\text{Small Number}}{\text{Small Number}}}_{\text{We cannot tell (Indeterminate)}} \overset{\text{Examples}}{\Longrightarrow} \frac{0.00003}{0.000001} = 30$ or $\frac{0.00003}{0.0000001} = 300$

② $\frac{a-b}{b-a} = \boxed{-1}$ because $\boxed{-(-b+a) = -(a-b)}$

Ex3| Find $\lim_{x\to 3} \frac{x^3-27}{9-x^2}$.

$\frac{27-27}{9-9} = \frac{0}{0}$ undefined

Solution:

$$\lim_{x\to 3} \frac{\cancel{(x-3)}^{-1}(x^2+3x+9)}{\cancel{(3-x)}(3+x)} = \lim_{x\to 3} \frac{-(x^2+3x+9)}{3+x} = \frac{-(9+9+9)}{3+3} = \frac{-27}{}$$

$$= \boxed{-45}$$

Ex4| Find $\lim_{x\to 2} \frac{(x^2+1)^2 - x^4 - 9}{3x-6}$.

$\frac{(4+1)^2 - 16 - 9}{6-6} = \frac{0}{0}$

Solution:

$$\lim_{x\to 2} \frac{\cancel{x^4}+2x^2+1-\cancel{x^4}-9}{3x-6} = \lim_{x\to 2} \frac{2x^2-8}{3x-6} = \lim_{x\to 2} \frac{2(x^2-4)}{3(x-2)}$$

$$= \lim_{x\to 2} \frac{2\cancel{(x-2)}(x+2)}{3\cancel{(x-2)}} = \lim_{x\to 2} \frac{2(x+2)}{3} = \frac{2(2+2)}{3} = \frac{2(4)}{3} = \boxed{\frac{8}{3}}$$

Ex5| Find $\lim_{x\to 0} \frac{\sqrt{2x+4}-2}{x}$.

$\frac{\sqrt{0+4}-2}{0} = \frac{2-2}{0} = \frac{0}{0}$

Solution:

Conjugate

$$\lim_{x\to 0} \frac{\sqrt{2x+4}-2}{x} \cdot \boxed{\frac{\sqrt{2x+4}+2}{\sqrt{2x+4}+2}} = \lim_{x\to 0} \frac{2x+\cancel{4}-\cancel{4}}{(\sqrt{2x+4}+2)x} =$$

$$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{2x+4}+2)} = \frac{2}{\sqrt{0+4}+2} = \frac{2}{2+2} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

Ex6| Find $\lim_{x\to -3} \frac{\frac{1}{x+5} - \frac{1}{2}}{2x+6}$

$\frac{0}{0}$

Solution:
We use Least Common Denominator (LCD)

$$\frac{1}{x+5} - \frac{1}{2} = \frac{2-(x+5)}{2(x+5)} = \frac{2-x-5}{2(x+5)} = \boxed{\frac{-x-3}{2(x+5)}}$$

$\Longrightarrow$

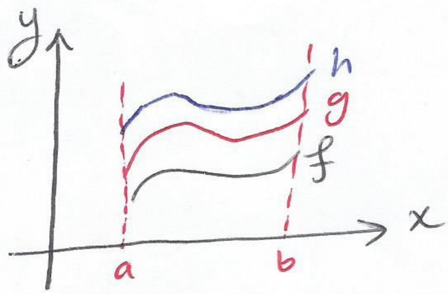
$$\Rightarrow \lim_{x\to -3} \frac{\frac{-x-3}{2(x+5)}}{2x+6} = \lim_{x\to -3} \frac{-x-3}{2(x+5)} \cdot \frac{1}{2x+6} =$$

$$= \lim_{x\to -3} \frac{-x-3}{2(x+5)(2x+6)} = \lim_{x\to -3} \frac{-\cancel{(x+3)}}{2(x+5)(2)\cancel{(x+3)}} =$$

$$= \lim_{x\to -3} \frac{-1}{4(x+5)} = \frac{-1}{4(-3+5)} = \frac{-1}{4(2)} = \boxed{\frac{-1}{8}}$$

* Theorem: Sandwich (Squeeze) Theorem

Suppose that $f(x) \leq g(x) \leq h(x)$ for all x in the domain, say D, and if $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then: $\lim_{x\to a} g(x) = L$.



Ex7 | Given $\cos x \leq f(x) \leq x^2+1$. Find: $\lim_{x\to 0} f(x) = ?$

Solution:

$$\lim_{x\to 0} \cos x \leq \lim_{x\to 0} f(x) \leq \lim_{x\to 0} x^2+1$$

$\lim_{x\to 0} \cos x = \cos 0 = 1$

$\lim_{x\to 0} x^2+1 = 0^2+1 = 1$

Thus, $\lim_{x\to 0} f(x) = 1$ by Sandwich Theorem