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Exponential and Logarithmic Functions

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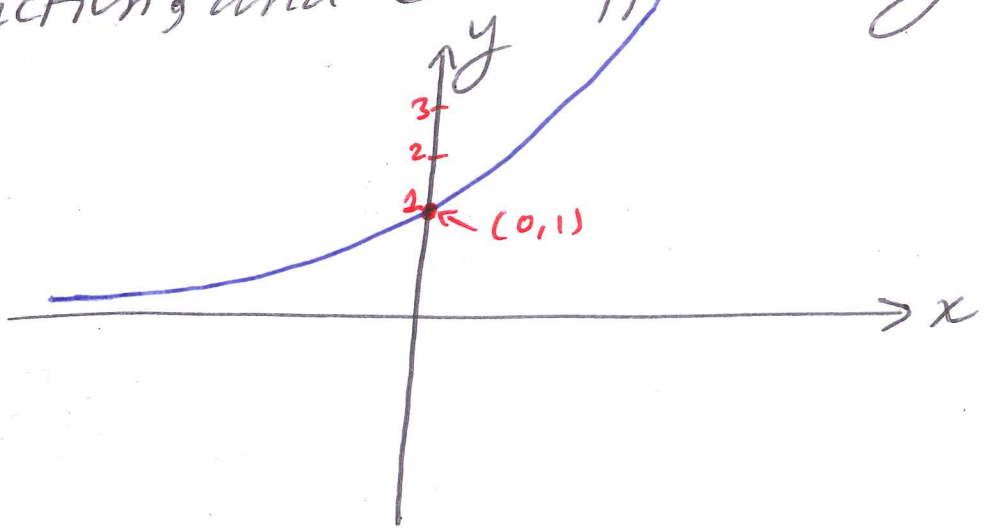
*Exponential Functions

Definition: Assume that we have a function: $f(x)=a^x$ where $a>0$ and $a\neq 1$. Then, exponential function is defind as follows: For each value of the constant a, known as the base, the exponent x may be assumed to be any real value. The domain of f is the set of all real numbers, and the range of f is the set of all positive numbers.

*Note: The most commen example of exponential function is: $f(x)=e^x$ where e is the base of function, and e is approximately ≈ 2.718281828



$$y=e^x$$

$$y\Big|_{x=0}=e^0=1$$

$$y\Big|_{x=1}=e^1\approx 2.718281828$$

Exponential and Logarithmic Functions

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* How to graph the exponential function: $f(x)=a^x$?

Answer: The graph should have the following properties:

Graph Properties

① The graph is continuous, with no breaks, holes, or jumps

② the x-axis is a horizontal asymptote

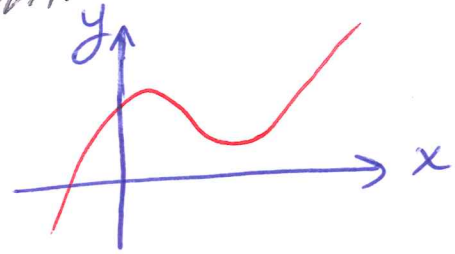
③ f is increasing if $a>1$

④ f is decreasing if $0<a<1$

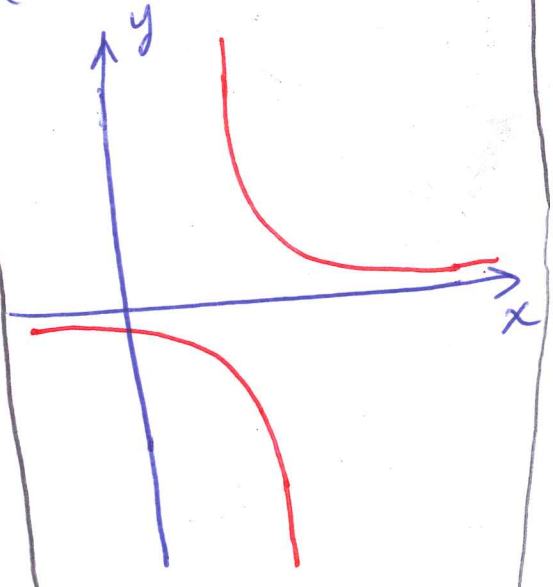
⑤ Since $a^0=1$, then this implies that the graph passes through the point $(0,1)$

⑥ f is one-to-one

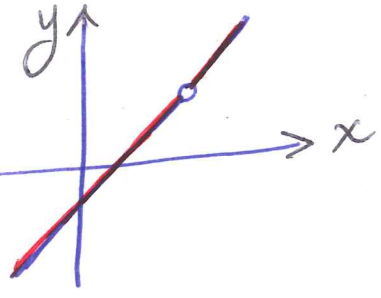
Continuous Function



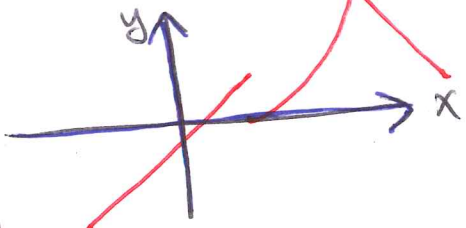
Break (Infinite Break)



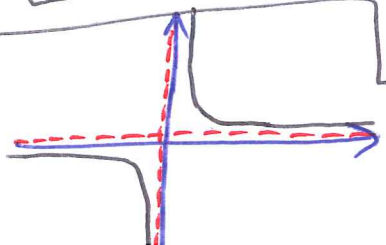
Hole



Jump



Asymptote ⟹



→ a line or curve that approaches a given curve arbitrarily closely.

* One-to-One Function: is defined as follows:

A function is one-to-one, denoted by 1-1, if

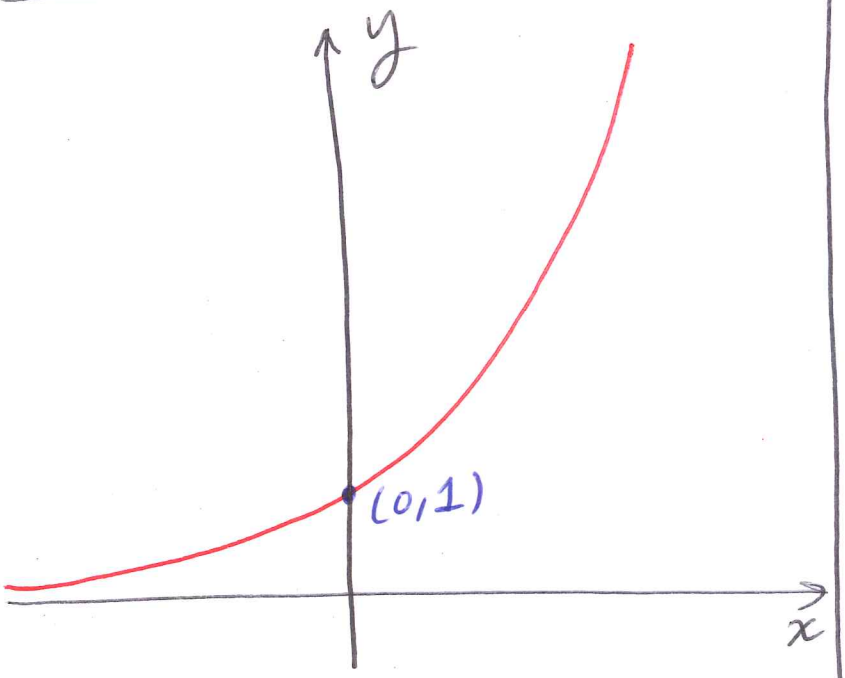
$$x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$

OR

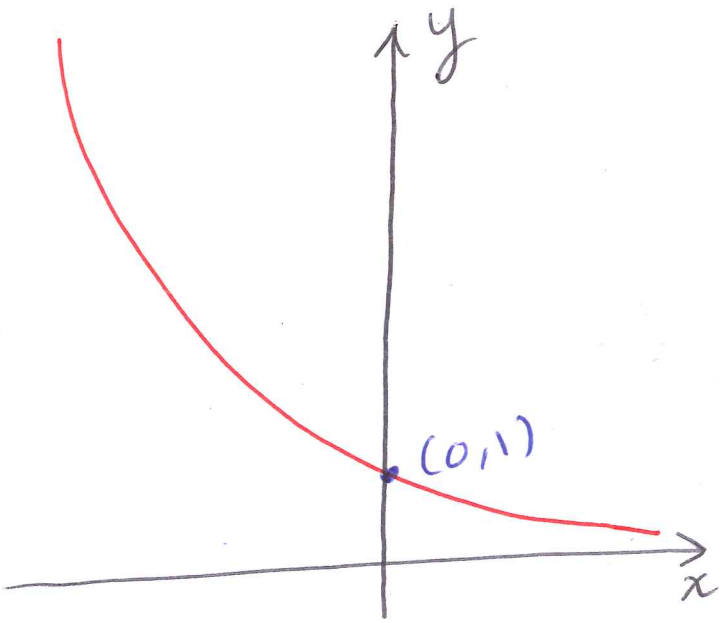
$$f(x_1) = f(x_2) \implies x_1 = x_2$$

* Two common cases of the graph of exponential function: $f(x) = y = a^x$

Case 1: $y = a^x$ where $a > 1$



Case 2: $y = a^x$ for $0 < a < 1$



*There are many applications of exponential functions:

- Exponential Growth
  - Biology
  - Economics
  - Compound Interest
    - P: is a principal that invested at an annual interest rate r.
    - r: Annual interest rate
    - A: the amount in the balance after t years
    - For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
    - For continuous compounding: $A = Pe^{rt}$
- Radioactive Decay
  - Physics
  - Chemistry

* Logarithmic Functions

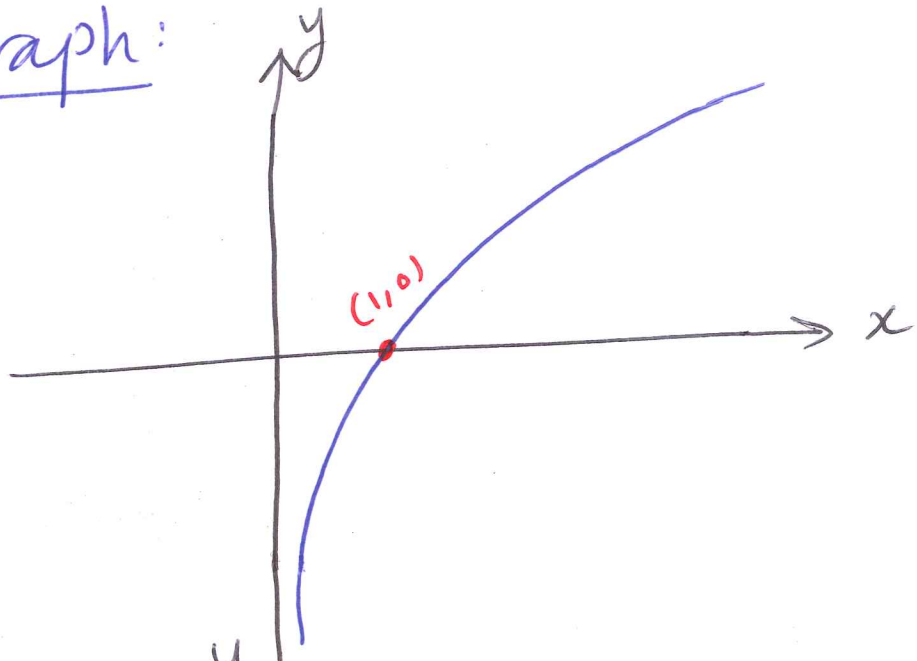
Definition: Assum $a>0$ and $a\neq 1$, the logarithmic function with a base a, written as $\log_a$, can be defined as follows:

$$y=\log_a x \iff a^y=x$$

(if and only if)

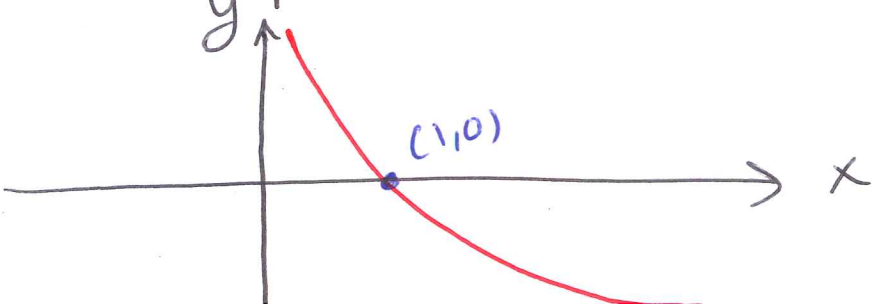
The domain of a logarithmic function is the set of all positive real numbers, and the range is the set of all real numbers.

Graph:



Case 1:

$y=\log_a x$ where $a>1$



Case 2:

$y=\log_a x$ for $0<a<1$

* Types of Logarithmic Functions

Common Logarithm

$$\log x = \log_{10} x$$

Natural Logarithm

$$\ln x = \log_e x$$

* Exponential Functions -Vs- Logarithmic Function

$$\log x = y \iff x = 10^y$$

$$\ln x = y \iff x = e^y$$

* Applications of Logarithmic Functions

- Exponential Growth
- Radioactive Decay
- Intensity of Earthquakes
- Loudness of Sounds