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Practice with Exponential & Logarithmic Functions

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*Example 1: Change each logarithmic form to the equivalent exponential form:

Part a: $\log_5 125 = 3$

Part b: $\log_{49} 7 = \frac{1}{2}$

Part c: $\log_2 \left(\frac{1}{8}\right) = -3$

Solution:

Part a: $\log_5 125 = 3 \Longleftrightarrow \boxed{5^3 = 125}$

Part b: $\log_{49} 7 = \frac{1}{2} \Longleftrightarrow 49^{\frac{1}{2}} = 7 \Rightarrow \boxed{\sqrt{49} = 7}$

Part c: $\log_2 \left(\frac{1}{8}\right) = -3 \Longleftrightarrow \boxed{2^{-3} = \frac{1}{8}} \Rightarrow \frac{1}{2^3} = \frac{1}{8}$

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*Example 2: Change each exponential form to the equivalent logarithmic form:

Part a: $\sqrt{81} = 9 \Rightarrow 81^{1/2} = 9$

Part b: $\frac{1}{2} = 8^{-\frac{1}{3}}$

Part c: $4^3 = 64$

Solution:

Part a:

$\sqrt{81} = 9 \Rightarrow 81^{1/2} = 9 \overset{\text{Equivalent}}{\Longleftrightarrow} \boxed{\log_{81} 9 = \frac{1}{2}}$

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Part b: $\frac{1}{2} = 8^{-\frac{1}{3}} \Longleftrightarrow \boxed{\log_8\left(\frac{1}{2}\right) = \frac{-1}{3}}$

Part c: $4^3 = 64 \Longleftrightarrow \boxed{\log_4 64 = 3}$

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*Example 3: Use logarithmic properties to write each expression in terms of simpler logarithmic forms:

Part a: $\log_a\left(\frac{xy}{5z}\right)$

Part b: $\log_a\left(\sqrt[3]{\frac{x^2}{3y}}\right)$

Solution:

Part a: $\log_a\left(\frac{xy}{5z}\right) = \log_a(xy) - \log_a(5z)$

$= \log_a(x) + \log_a(y) - \left(\log_a(5) + \log_a(z)\right)$

$= \log_a(x) + \log_a(y) - \log_a(5) - \log_a(z)$

Part b: $\log_a\left(\sqrt[3]{\frac{x^2}{3y}}\right) = \log_a\left(\frac{x^2}{3y}\right)^{1/3} = \frac{1}{3}\log_a\left(\frac{x^2}{3y}\right)$

$= \frac{1}{3}\left[\log_a(x^2) - \log_a(3y)\right]$

$= \frac{1}{3}\left[2\log_a x - \log_a 3 - \log_a y\right]$

$= \boxed{\frac{2}{3}\log_a x - \frac{1}{3}\log_a 3 - \frac{1}{3}\log_a y}$

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*Example 4: Write an expression (Single Expression) for the following:

$5\ln(x) + \frac{1}{2}\ln(x) - 3\ln(x+1)$

Solution:

$5\ln(x) + \frac{1}{2}\ln(x) - 3\ln(x+1) = \ln x^5 + \ln x^{\frac{1}{2}} - \ln(x+1)^3$

$= \ln\left[\frac{x^5 x^{\frac{1}{2}}}{(x+1)^3}\right] = \ln\left[\frac{x^5\sqrt{x}}{(x+1)^3}\right]$

*Example 5: Find the domain of each logarithmic function:

Part a: $f(x) = \log_4(9-x^2)$

Part b: $f(x) = \ln(x) + \ln(3-x)$

Solution:

Part a: Since $\log_a x$ is defined when $x>0$, then the domain of function is as follows: $9-x^2>0$

$\sqrt{9} > \sqrt{x^2}$

$3 > |x|$

So, the domain is $(-3,3)$.

Part b: $\ln(x)$ is defined for $x>0$. So, for $\ln(3-x)$, we obtain: $3-x>0 \Rightarrow 3>x$

Thus, the domain is $x>0$ and $x<3 \Rightarrow (0,3)$.