



Handout 7

MATH 140 Lab: Section 1

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Student's Name:-----

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Note: This handout covers the most important things that can help you beat the difficulty of Math 140 exam 1.

- When you evaluate the limit and you get $\frac{0}{0}$, then **PLEASE TRY TO USE ONE OF THE FOLLOWING:**
 - a. Factorization
 - b. Simplification
 - c. Least Common Denominator
 - d. Conjugate
 - e. Sandwich (Squeeze) Theorem
- When you evaluate the limit and you get $\frac{\infty}{\infty}$, then **PLEASE USE LEADING TERMS METHOD TO FIND THE LIMIT.** Note: Leading terms method is a method that we take the limit of the highest degree term (**COEFFICIENT + VARIABLE**) in **numerator** divided by the highest degree term (**COEFFICIENT + VARIABLE**) in **denominator**.
- Please remember the following when you find the limit:
 - a. $\infty + \infty = \infty$
 - b. $\infty \cdot \infty = \infty$
 - c. $\infty - \infty = \text{Indeterminate}$ (PLEASE be careful: $\infty - \infty$ **does NOT equal** a **zero**)
 - d. $\frac{\infty}{\infty} = \text{Indeterminate}$ (PLEASE be careful: $\frac{\infty}{\infty}$ **does NOT** a **one**)
 - e. $\frac{1}{\infty} = 0$ (**Note: Any non-zero number over infinity is zero**)
 - f. $\frac{1}{0} = \infty$ (**Note: Any non-zero number over zero is infinity**)
- To check your answer (Finding Limit) to see whether it correct or incorrect, there is one UNOFFICIAL WAY for that: Use *L'Hôpital's Rule* (I will show this way in class during our review session) **PLEASE DO NOT USE IT AS YOUR OFFICIAL SOLUTION TO FIND THE LIMIT,** BUT IT IS RECOMMENDED TO USE IT TO **CHECK YOUR SOLUTION ONLY.**

For example: Find $\lim_{x \rightarrow 1} \left(\frac{2x-2}{x-1} \right)$.

Solution: L'Hôpital's Rule:

- 1- Find the derivative for Numerator(The derivative of $(2x - 2)$ is 2).
- 2- Find the derivative for Denominator(The derivative of $(x - 1)$ is 1).
- 3- $\lim_{x \rightarrow 1} \left(\frac{2x-2}{x-1} \right) = \lim_{x \rightarrow 1} \left(\frac{2}{1} \right) = \frac{2}{1} = 2.$

- There cases where you have to find **the limit from both sides (Left-Hand Side and Right-Hand Side):**
 - a. Roots
 - b. Absolute Value Functions
 - c. Piecewise-Defined Functions
 - d. $\frac{1}{0} = \pm\infty$
- If the question in the exam says: Find the **derivative using the definition of derivative,** then **PLEASE PLEASE PLEASE use the following:**

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

Note: In this case: **PLEASE DO NOT USE THE SHORT-CUT WAY TO FIND THE DERIVATIVE.**

- If the question in the exam says: Find the **derivative using the definition of derivative at $x = a$,** then **PLEASE PLEASE PLEASE use either of the following:**

$$\bullet f'(a) = \lim_{h \rightarrow 0} \left(\frac{f(a+h) - f(a)}{h} \right)$$

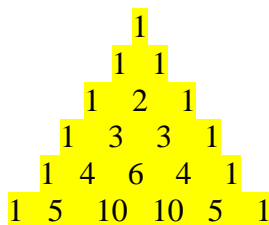
OR

$$\bullet f'(a) = \lim_{x \rightarrow a} \left(\frac{f(x) - f(a)}{x - a} \right)$$

Note: In this case: **PLEASE DO NOT USE THE SHORT-CUT WAY TO FIND THE DERIVATIVE at $x = a$.**

- If you see something like $(x + y)^3$, $(x + y)^4$, or $(x - y)^3$ and you need to simplify it, then please use Pascal's Triangle Method to simplify that:

Pascal Triangle Method for Simplification:



Examples:

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$(x - y)^4 = (x + (-y))^4$$

- If you need to find the equation of tangent line, then please remember that the derivative at the given point is the slope of tangent line, and use the following equation:

$$y - y_1 = m(x - x_1) \text{ where } (x_1, y_1) \text{ is the given point.}$$

- PLEASE REMEMBER THE FOLLOWING IMPORTANT **DIFFERENTIATION RULES AND THEOREMS:**

$$\textcircled{1} (c)' = 0$$

$$\textcircled{2} (x^n)' = nx^{n-1}$$

$$\textcircled{3} (cf(x))' = cf'(x)$$

$$\textcircled{4} (f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\textcircled{5} \text{ Product Rule: } (f(x) \cdot g(x))' = f'(x)g(x) + g'(x)f(x)$$

$$\textcircled{6} \text{ Quotient Rule: } \left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\textcircled{7} \text{ Chain Rule: } (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

*Differentiation Theorems:

$$\textcircled{1} (e^x)' = e^x$$

$$\textcircled{2} (\ln x)' = \frac{1}{x}$$

Generally,

$$(e^{\square})' = e^{\square} \cdot \square'$$

$$(\ln \square)' = \frac{1}{\square} \cdot \square'$$

*General Rules for Differentiation:

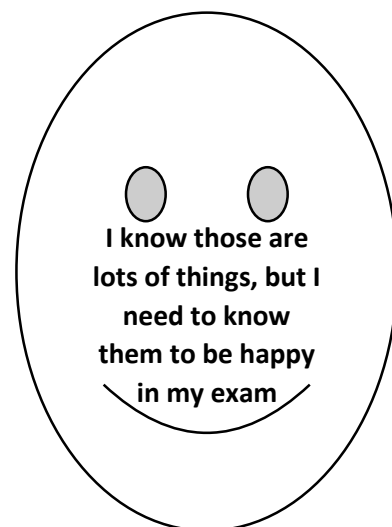
$$(a^x)' = a^x \ln a$$

$$\left(\log_a x\right)' = \frac{1}{x \ln a}$$

- PLEASE REMEMBER THE FOLLOWING **IMPORTANT THINGS:**

Some Important Simplifications:

- $A^2 - B^2 = (A - B)(A + B)$
- $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
- $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
- $(A - B)^2 = (A^2 - 2AB + B^2)$
- $A^4 - B^4 = (A^2 - B^2)(A^2 + B^2) = (A - B)(A + B)(A^2 + B^2)$
- $\frac{A-B}{B-A} = -1$



Derivatives:

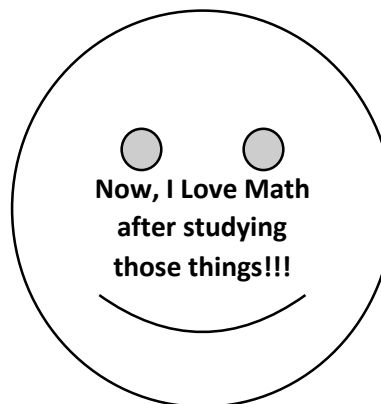
- 1- The derivative of $\sin(x)$ is: $\cos(x)$
- 2- The derivative of $\cos(x)$ is: $-\sin(x)$
- 3- The derivative of $\tan(x)$ is: $\sec^2 x$
- 4- The derivative of $\cot(x)$ is: $-\csc^2 x$
- 5- The derivative of $\sec(x)$ is: $\sec(x) \tan(x)$
- 6- The derivative of $\csc(x)$ is: $-\csc(x) \cot(x)$
- 7- The derivative of $\ln(x)$ is: $\frac{1}{x}$
- 8- The derivative of e^{2x} is $2e^{2x}$

Simplifications:

- 1- $\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$
- 2- $\sin(A + B) = \sin(A) \cos(B) + \sin(B) \cos(A)$

Trigonometric Functions:

- 1- $\sin(0) = 0$
- 2- $\cos(0) = 1$
- 3- $\sin(90) = 1$
- 4- $\cos(90) = 0$
- 5- $\sin(180) = 0$
- 6- $\cos(180) = -1$
- 7- $\sin(360) = 0$
- 8- $\cos(360) = 1$
- 9- $\sin(30) = \frac{1}{2}$
- 10- $\cos(60) = \frac{1}{2}$
- 11- $\tan(45) = 1$
- 12- $\tan(90) = \text{Undefined}$
- 13- $\sin(45) = \cos(45) = \frac{\sqrt{2}}{2}$

**Good Luck in MATH 140 Exam 1 and Read the following Advice:**

- **Consider exam 1 as a mathematical game rather than a challenging thing in your life.**
- **Do not get scared from Math because Math is not a zombie and nothing is scary in Math.**
- **Study Everything Kaabar gave you + Study Lecture Materials + Study all Sample Exams and Study Guide → Pass Exam 1 + Get an "A" + Become Happy with Math + Maybe interested in taking a Minor in Mathematics (I hope so!!)**
- **Need to ask any question, PLEASE PLEASE PLEASE contact me as soon as possible.**
- **Got bored from studying Math 140, then I would leave it and do something else such as listening to music, watching a movie, hiking, fishing, or driving off-roads and on snowy roads (I do these things when I get bored from studying my Math courses)**