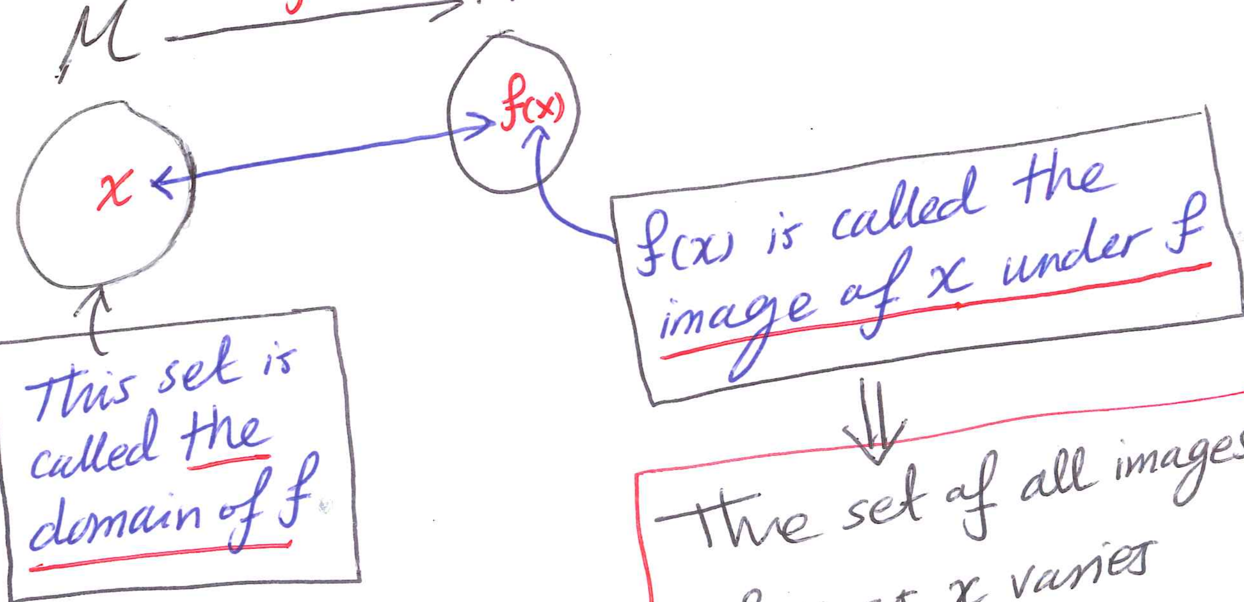
Mohammed Kaabar   Calculus for Life Scientists   Jan. 13th, 2016

Review

* Functions:

Definition: Suppose that we have two sets say M and N. Then, a function f from a set M to a set N is a rule that assigns to each element x in the set M exactly one element $f(x)$ in the set N.

$$M \xrightarrow{f} N$$



The set of all images $f(x)$ as x varies throughout the domain is called the range of f

Example

$x = 0 \longrightarrow f(0)$

$x = 1 \longrightarrow f(1)$

$x = 2 \longrightarrow f(2)$

$\vdots \qquad \vdots$

$x = ? \longrightarrow f(?)$

Range

Review

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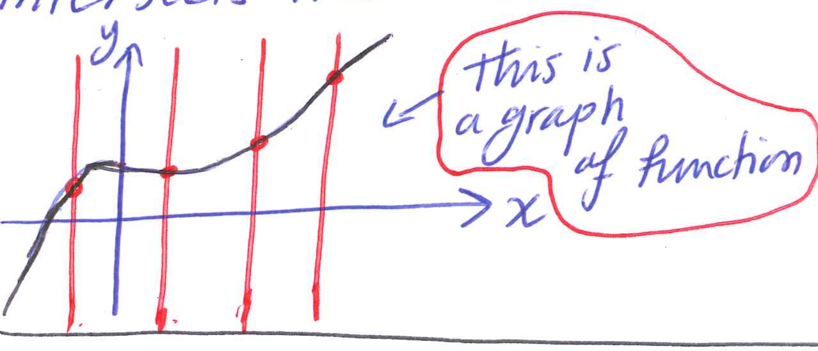
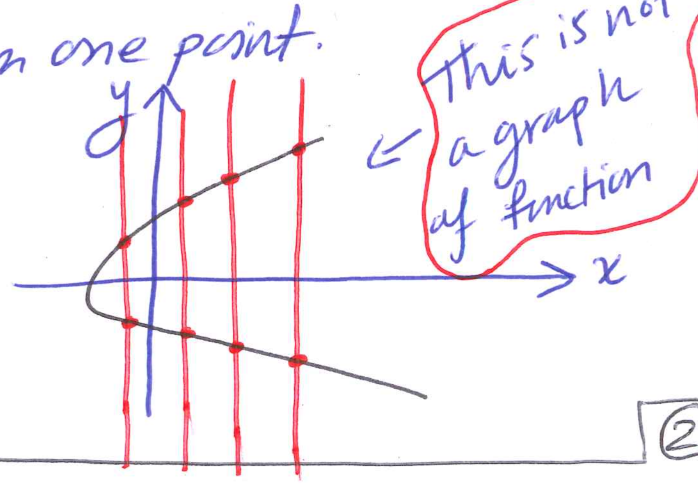
* How to represent functions?

<u>Answer:</u>

- **Functions**
  - Numerical Representation
    - Table
    - Set of ordered pairs
  - Geometric Representation
    - Graph
  - Algebraic Representation
    - Formula

* <u>The Vertical Line Test:</u> is a test used to determine whether a given curve is the graph of a function or not. This test says that a curve in the xy-plane is the graph of a function iff no vertical line intersects the curve more than one point.





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* **Combinations of Functions**; Given: f and g are functions

- **Sum**

$(f+g)(x) = f(x) + g(x)$

- **Difference**

$(f-g)(x) = f(x) - g(x)$

- **Product**

$(fg)(x) = f(x) \cdot g(x)$

- **Quotient**

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$

* **Composite Functions**

Definition: The composition of functions f and g, denoted by $f \circ g$ can be written as follows:

$(f \circ g)(x) = f(g(x))$

- **Note**: the domain of composite function $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is the domain of f.

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* Piecewise Defined Functions

Definition: Functions are sometimes defined using different formulas on different parts of its domain.

Ex 1:

$$f(x) = \begin{cases} 7x+1 & , \quad x<4 \\ x-3 & , \quad 4 \le x < 8 \\ -2 & , \quad x = 5 \\ 3x+7 & , \quad x=4 \\ 4x & , \quad x>8 \end{cases}$$

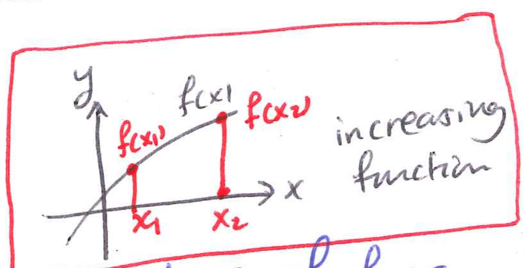
Ex 2: Absolute Value Function: $|x| = \begin{cases} x & , \quad x \geq 0 \\ -x & , \quad x<0 \end{cases}$

is also a piecewise defined function.

* Even and Odd Functions:

* The function f is even if $\boxed{f(-x)=f(x)}$ for all x in the domain of f.
* The function f is odd if $\boxed{f(-x)=-f(x)}$ for all x in the domain of f.

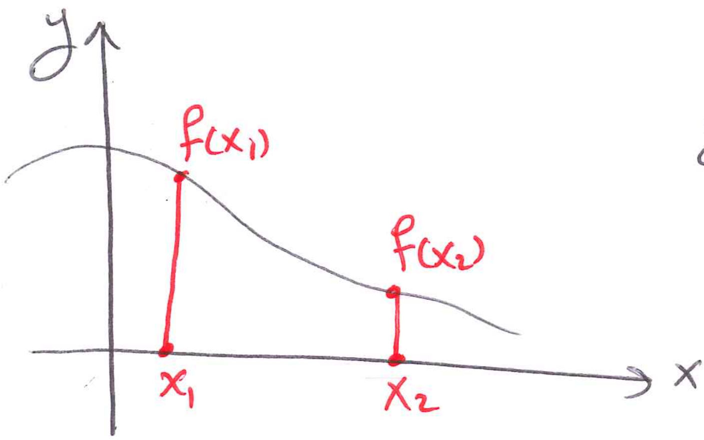
* Increasing and Decreasing Functions



* The function f is increasing on an interval I if for any x_1 and x_2 in I, $\boxed{x_1 < x_2} \overset{\text{implies}}{\Longrightarrow} \boxed{f(x_1) < f(x_2)}$

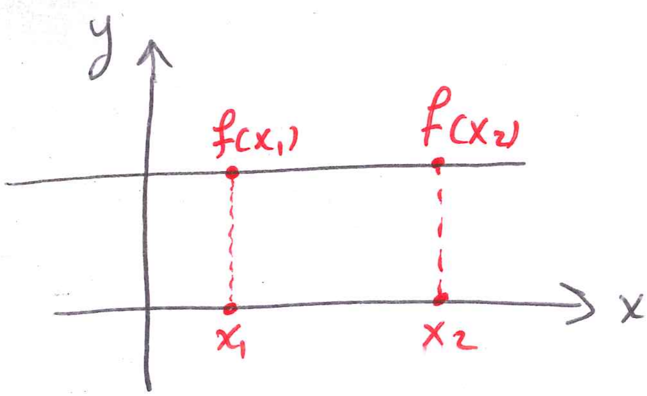
* The function f is decreasing on an interval I if, for any x_1 and x_2 in I,

$$\boxed{x_1 < x_2} \xRightarrow{\text{implies}} \boxed{f(x_1) > f(x_2)}$$



decreasing function

* The function f is constant on an interval I, if for any x_1 and x_2 in I, $\boxed{f(x_1) = f(x_2)}$



Example 1: Given: $f(x) = -2x^2 - 2$ and $g(x) = \sqrt{2x+3}$

Find the following:

ⓐ $f(-3)$

ⓑ $f(2t)$

ⓒ $\dfrac{f(a+h) - f(a)}{h}$

ⓓ $g(4)$

Solution:

Part a: $f(-3) = -2(-3)^2 - 2 = -2(9) - 2 = -18 - 2 = \boxed{-20}$

Part b: $f(2t) = -2(2t)^2 - 2 = -2(4t^2) - 2 = \boxed{-8t^2 - 2}$

Part c: $\dfrac{f(a+h) - f(a)}{h}$

$f(a) = -2a^2 - 2$

$f(a+h) = -2(a+h)^2 - 2$

$$\frac{f(a+h)-f(a)}{h} = \frac{-2(a+h)^2 - 2 - (-2a^2 - 2)}{h}$$

$$= \frac{-2(a+h)^2 \cancel{-2} + 2a^2 \cancel{+2}}{h}$$

$$= \frac{-2(a^2 + 2ah + h^2) + 2a^2}{h}$$

$$= \frac{\cancel{-2a^2} - 4ah - 2h^2 \cancel{+2a^2}}{h}$$

$$= \frac{-4ah - 2h^2}{h} = \frac{-2\cancel{h}(2a+h)}{\cancel{h}}$$

$$= -2(2a+h) = \boxed{-4a - 2h}$$

Part d: $g(4) = \sqrt{2(4)+3} = \sqrt{8+3} = \sqrt{11}$

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Example 2: Find the domain of each function:

ⓐ $f(x) = x^2 + 2x - 3$

ⓑ $f(x) = \sqrt{6-2x}$

Solution:

Part a: The function is defined for all values of x. This implies that the domain is all real numbers

Domain $= (-\infty, \infty)$

Part b: Since it's impossible to take the square root of a negative numbers, then we need to do the following:

$6 - 2x \geq 0$

$\frac{6}{2} \geq \frac{2x}{2}$

$3 \geq x \Rightarrow \boxed{x \leq 3}$

Domain: $(-\infty, 3]$

Example 3: Determine whether each function is even, odd, or neither:

ⓐ $f(x) = |x|$

ⓑ $f(x) = \cos(x)$

Part a: $f(-x) = |-x| = |-1||x| = |x| = f(x)$

So, it's <u>even</u>.

Part b: $f(-x) = \cos(-x) = \cos(x) = f(x)$

So, it's <u>even</u>.

* Applied Functions: Variation (Proportionality):

- The quantity y is directly proportional to x if there exists a constant $k \neq 0$ (proportionality constant) such that $\boxed{y = kx}$.
- The quantity y is inversely proportional to x if there exists a constant $k \neq 0$ (proportionality constant) such that $\boxed{y = \frac{k}{x}}$
- The quantity z is jointly proportional to x and y if there exists a constant $k \neq 0$ (proportionality constant) such that $\boxed{z = kxy}$