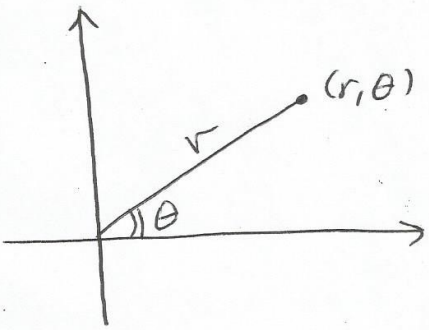
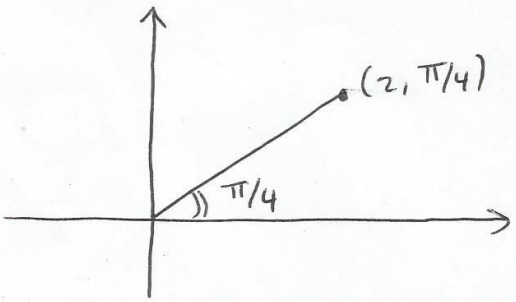
* Polar Coordinates (r, θ)

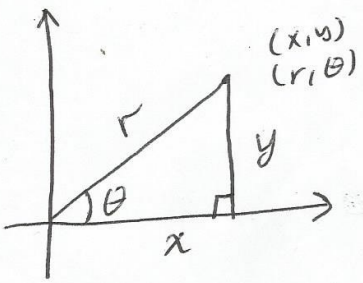


Example ①: Draw the polar coordinates for $(2, \pi/4)$.

Solution



* The relationship between polar coordinates (r, θ) and Cartesian coordinates (x, y):



$$\begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \end{cases}$$

$$\begin{cases} \sin\theta = \frac{y}{r} \\ \cos\theta = \frac{x}{r} \end{cases}$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

Example ②: Change from polar coordinate to cartesian coordinate : $(r, \theta) = (4, \frac{\pi}{4})$

Solution:

$$x = r\cos\theta = 4\cos\left(\frac{\pi}{4}\right) = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

$$y = r\sin\theta = 4\sin\left(\frac{\pi}{4}\right) = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}$$

So, $(x, y) = (2\sqrt{2}, 2\sqrt{2})$

Example ③: Change from cartesian coordinate to polar coordinate: $(x, y) = (5, 5)$

Solution:

$$r^2 = x^2 + y^2$$

$$r^2 = 25 + 25 = 50 \Rightarrow \boxed{r = \sqrt{50}}$$

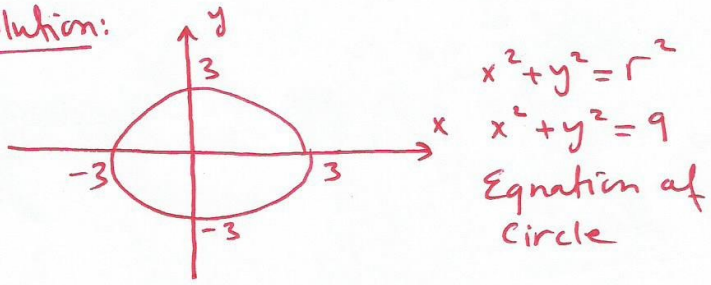
$$\tan\theta = \frac{y}{x} = \frac{5}{5} = 1 \Rightarrow \theta = \tan^{-1}(1) = \frac{\pi}{4}$$

So, $(r, \theta) = (\sqrt{50}, \frac{\pi}{4})$

Example ④: Draw the following:

Part a: $r = 3$

Solution:



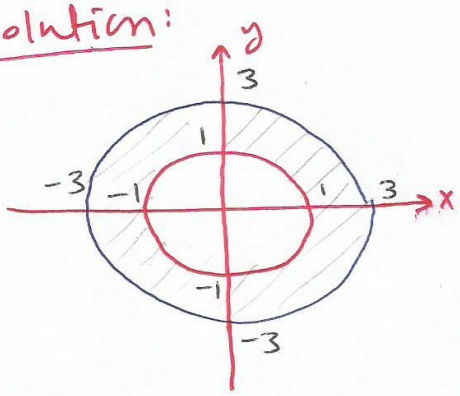
$x^2 + y^2 = r^2$

$x^2 + y^2 = 9$

Equation of circle

Part b: $1 \leq r \leq 3$

Solution:

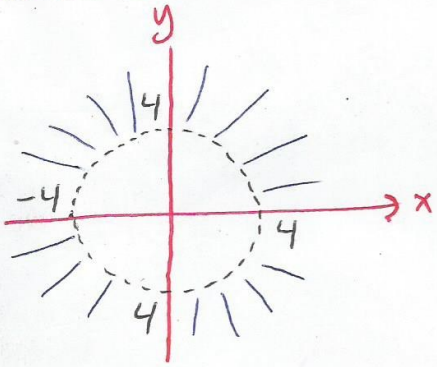


$x^2+y^2=1$ ← small circle

$x^2+y^2=9$ ← big circle

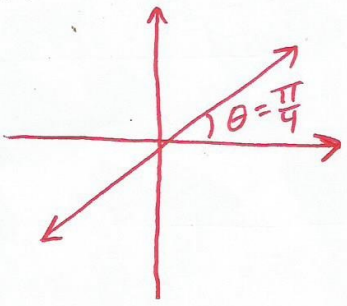
Part c: $r > 4$

Solution:



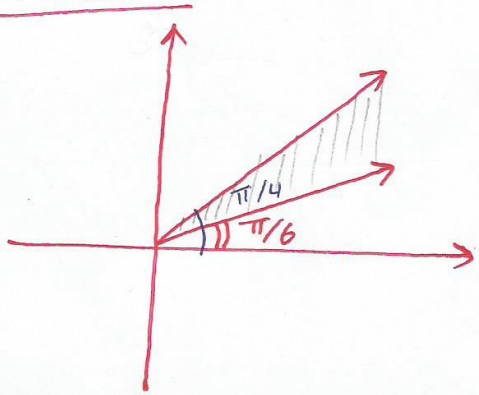
Part d: $\theta = \frac{\pi}{4}$

Solution:



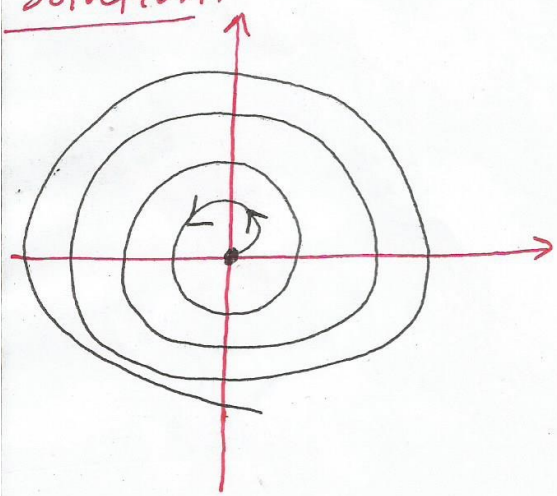
Part e: $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{4}$ and $r > 0$

Solution:



Part f: $r = \theta$, $\theta \geq 0$

Solution:



Spiral

Part g: $r = 3 + 2\cos\theta$

r	5	4.41	4	3	2	1
θ	0	π/4	π/3	π/2	2π/3	π

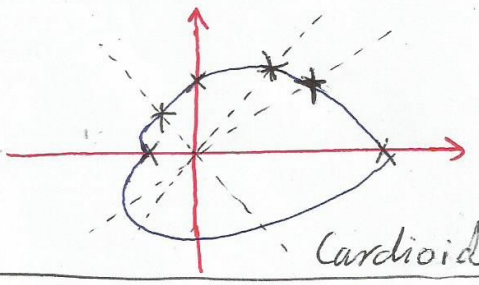
Solution

Replace (r, θ) by $(r, -\theta)$

$r = 3 + 2\cos(-\theta)$

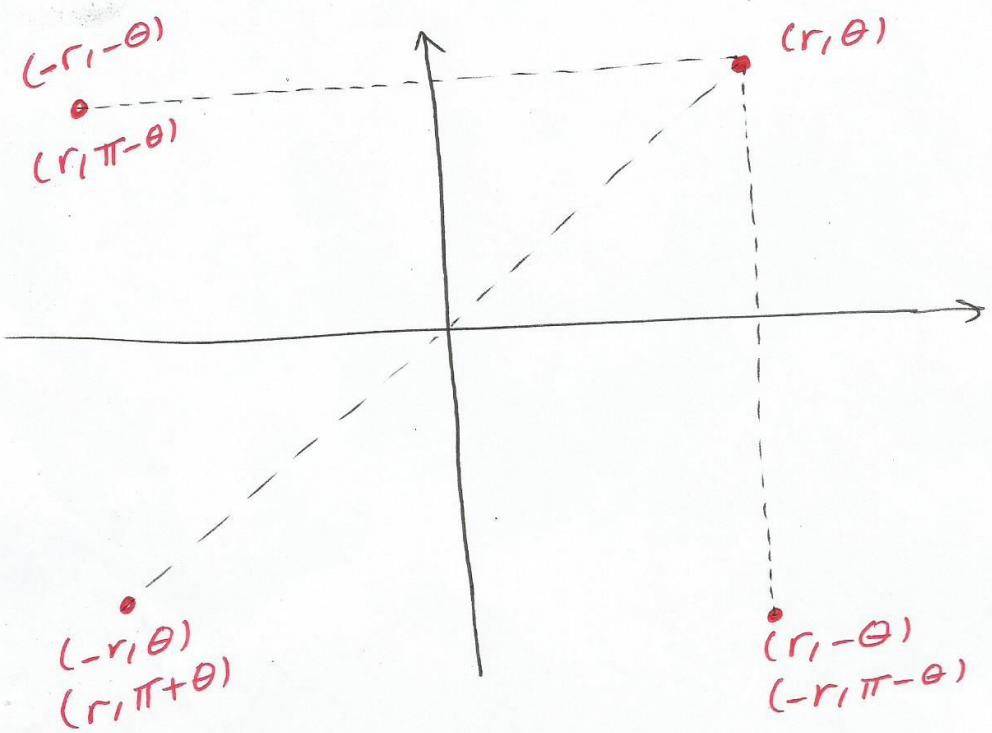
$r = 3 + 2\cos(\theta)$

Symmetric with respect to X-axis



Cardioid

* Symmetry Tests:



Example ⑤: Draw the following:

$r = 2 + 4\cos\theta$

Solution:

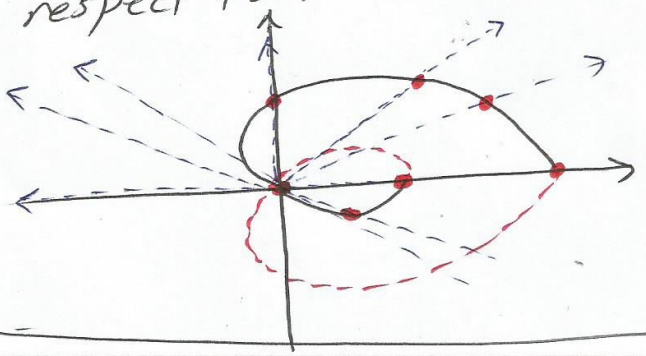
Replace (r, θ) by $(r, -\theta)$

$r = 2 + 4\cos(-\theta)$

$r = 2 + 4\cos(\theta)$

Symmetric with respect to x-axis

r	6	4.8	4	2	0	-0.8	-2
θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π



limaçon