Quiz 6

MATH 172 Lab: Sections 7

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Note: This quiz covers Alternating Series Test and Taylor Polynomials.

Show your work and circle your answers. Neatness and organization count!

Question 1: (3 points) Determine whether the following series diverges, converges conditionally, or converges absolutely:

$$\sum_{\varphi=1}^{\infty}(-1)^{\varphi-1}\frac{2^{\varphi}}{\varphi^{4}} \longrightarrow \text{So, it's } \boxed{\text{conditionally Convergent}}$$

Compare $\sum_{\varphi=1}^{\infty}(-1)^{\varphi-1}\frac{2^{\varphi}}{\varphi^{4}}$ with $\sum_{\varphi=1}^{\infty}\frac{2^{\varphi}}{\varphi^{4}}$

$$\lim_{\varphi\to\infty}\left|\frac{\cancel{2^{\varphi}}\,2^{1}}{(\varphi+1)^4}\cdot\frac{\varphi^4}{\cancel{2^{\varphi}}}\right| = \frac{2\varphi^4}{(\varphi+1)^4} = \frac{2\varphi^4}{\varphi^4} = 2 > 1 \text{ diverges}$$

by ratio test.

$$\boxed{\frac{d}{d\varphi}(2^{\varphi}) = 2^{\varphi}\ln(2)}$$

(i) Alternating Series

(ii) $\lim_{\varphi\to\infty}\frac{2^{\varphi}}{\varphi^4} \overset{L'H}{=} \frac{2^{\varphi}\ln(2)}{4\varphi^3} = \frac{2^{\varphi}\ln(2)\cdot\ln(2)}{12\varphi^2} = \frac{2^{\varphi}(\ln 2)^3}{24\varphi}$

$$= \frac{2^{\varphi}(\ln 2)^3}{\infty} = 0$$

(iii) Assume $f(x) = \frac{2^x}{x^4} = 2^x x^{-4} \Rightarrow f'(x) = (2^x\ln(2))x^{-4} + 2^x(-4x^{-5})$

$\Rightarrow f'(x) = \frac{2^x\ln(2)}{x^4} - \frac{2^x(4)}{x^5} < 0$ decreasing. So, it's convergent by Alternating Series Test.

Question 2: (2 points) Find the 5[th] degree Taylor Polynomial for $f(x) = \ln(1+x)$ centered at $x = 0$.

$f(x) = \ln(1+x) \longrightarrow f(0) = \ln(1+0) = 0$

$f'(x) = \frac{1}{1+x} = (1+x)^{-1} \longrightarrow f'(0) = 1$

$f''(x) = -(1+x)^{-2} \longrightarrow f''(0) = -1$

$f'''(x) = 2(1+x)^{-3} \longrightarrow f'''(0) = 2$

$f^{(4)}(x) = -6(1+x)^{-4} \longrightarrow f^{(4)}(0) = -6$

$f^{(5)}(x) = 24(1+x)^{-5} \longrightarrow f^{(5)}(0) = 24$

$\ln(1+x) \approx 0 + 1\cdot x - \frac{1\cdot x^2}{2!} + \frac{2\cdot x^3}{3!} - \frac{3!\, x^4}{4!} + \frac{4!\cdot x^5}{5!}$

$$\boxed{\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}}$$