

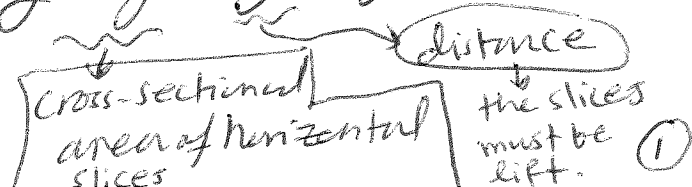
\* Def. Mass of 1-D Object: Assume a thin bar or wire is represented by the interval  $a \leq x \leq b$  with a density function  $\rho$  ( $\frac{\text{mass}}{\text{length}}$ ). Then, mass can be written as  $m = \int_a^b \rho(x) dx$ .

\* Def. of Work: the work done by a variable force  $F$  moving an object along a line from  $x=a$  to  $x=b$  in the direction of the force is written as  $W = \int_a^b F(x) dx$ .

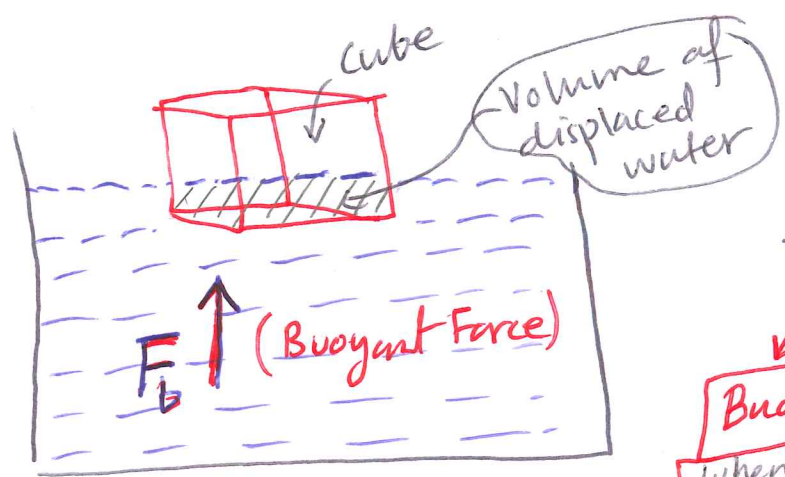
\* Remark: the work in (J's) required to lift an object of mass  $m$  a vertical distance of  $y$  can be written as  $W = F \cdot d = (mg)y$  where  $F$  is a gravitational force ( $F = mg$  where  $g \approx 9.8 \text{ m/s}^2$  the acceleration due to gravity near the surface of Earth).

$\downarrow$  Work       $\downarrow$  Force       $\downarrow$  distance

\* Note: To solve lifting problems, the work required to lift the water is  $W = \int_a^b \rho g A(y) D(y) dy$  where  $a \leq y \leq b$ .



Work Lab: Buoyancy and Archimedes' Principle



$$F_b = (\text{Volume Displaced}) \cdot \rho_w \cdot g$$

gravitational constant  $\downarrow$   
 $g$   
 density of water  $\downarrow$   
 $\rho_w$

**Buoyant Force**  
 when the object is totally submerged:

Archimedes' Principle: the buoyant force exerted on an object whether partially or totally submerged in water is equal to the weight of the water displaced by the object.

Some Useful Notes:

- ① The buoyant force increases as the cube is pushed further down below the water. This implies that the volume submerged increases as a result.
- ② Buoyant force stops increasing once cube is fully submerged. This implies that the buoyant force is proportional to volume submerged.

Let's discuss the three different cases for Archimedes principle.

I. If the density of water greater than object density, then the buoyant force is greater than the weight of the object. This implies that the object floats.

II. If the density of water equals object density, then the buoyant force equals the weight of the object. This implies that the object floats totally submerged.

III. If the density of water less than object density, then the buoyant force is less than the weight of the object. This implies that the object sinks.

Remark: If an object floats with a fraction, say  $f$ , where  $0 < f \leq 1$ , of its volume submerged, then  $\rho = f \rho_w$ . This implies that its density is the same fraction of the density of water.