QE Solutions

$ax^2+bx+c=0 \quad a\neq 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ to solve QE

QE formula

$\boxed{\Delta = b^2-4ac}$

discriminant

① If $\Delta = 0$, then the QE has one root (solution) (repeated root) or (double root)

② If $\Delta > 0$, then the QE has two Real Solutions (roots)

③ If $\Delta < 0$, then the QE has two imaginary (complex) solutions
not real $\sqrt{-?}$

* Rational Equations:-

Rational #s = $\frac{\text{integer}}{\text{non-zero integer}}$ $\frac{5}{3}, \frac{2}{5}$

* **Rational Expression:** is an expression that has a quotient of Polynomial $A(x)$ over another polynomial $B(x)$, say $\frac{A(x)}{B(x)}$.

Some Examples:-

① $\frac{x^2+2x+5}{3x+5}$

② $\frac{3}{2x+5}$

③ $\frac{x^3+2x^2+x+5}{x^2+3x+5}$

Polynomical General Form:-
$C_n x^n + C_{n-1} x^{n-1} + \dots + C_1 x + C_0$

$5x^3 + 2x^2 + x + 5x^0$

$5x^{100} + 4x^{99} + 2x^{98} + \dots + x + 2$

Simplify (Evaluate) the following:

LCD

$\frac{6x}{3x} = \boxed{2}$

$\frac{6x}{2x} = \boxed{3}$

$\frac{4-15}{6x} = \boxed{\frac{-11}{6x}}$

LCD

$= 0$

$= 0$

$= 0$

(3)

Ex 3) Solve :- $\boxed{\dfrac{5x}{2x+2} - \dfrac{1}{2} = 0}$

$$\downarrow$$

$$\frac{[10x - (2x+2)]}{2(2x+2)} = \frac{0}{1}$$

$$[10x - 2x - 2] = 0$$

$$10x - 2x - 2 = 0$$

$$8x - 2 = 0$$

$$8x = 2$$

$$x = \frac{\cancel{2}^{1}}{\cancel{8}4} \Rightarrow$$

Solution

$$\boxed{x = \frac{1}{4}}$$

$\boxed{\begin{aligned} a &= 1 \\ b &= -1 \\ c &= -6 \end{aligned}}$

$\sqrt{25} = 5$ $\sqrt{9} = 3$

$\sqrt{16} = 4$ $\sqrt{64} = 8$

$\sqrt{49} = 7$

$\sqrt{81} = 9$

QE Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{1 - 4(1)(-6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 + 24}}{2}$$

$$= \frac{1 \pm \sqrt{25}}{2}$$

$$= \frac{1 \pm 5}{2}$$

$\boxed{\begin{aligned} &x = 3 \\ &\text{or} \\ &x = -2 \end{aligned}}$

QE Formula $x^2 - x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -1$
$c = -6$

Solution

$$x = \frac{1 \pm \sqrt{1 - 4(1)(-6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 + 24}}{2}$$

$$= \frac{1 \pm \sqrt{25}}{2}$$

$$= \frac{1 \pm 5}{2}$$

solution

$\boxed{x = 3}$ or $\boxed{x = -2}$

Ex2) Solve the following :-

$$\sqrt{2x-3} + x = 1$$

$$\left(\sqrt{2x-3}\right)^2 = (1-x)^2$$

$(a-b)^2 =$
$a^2 - 2ab + b^2$

$$2x - 3 = 1 - 2x + \boxed{x^2}$$

$$x^2 - 4x + 4 = 0$$

$x \quad -2 \longrightarrow (x-2)(x-2) = 0$

$x \quad -2$

$x = 2$ or $x = 2$

$-2x + -2x = -4x$

Radical Equations

$\sqrt[\boxed{3}]{}$ → root $\sqrt[4]{}$

$\boxed{=}\,0$

$\sqrt{2}$

Example:

① $x = \sqrt{x+7}$

② $\sqrt{3x-1} + \sqrt{2x+1} = x+5$

absolute value ~~problem~~ equation

$|2x+5| = 3$

$\emptyset \in$

poly: $= 0$

$x^2 + 2x + 5 = 0$

$\left(2^{\frac{1}{3}}\right)^{3} = 2$

$\sqrt[3]{2} \longrightarrow 2$

$\left(\sqrt[3]{2}\right)^3 = 2$

$\sqrt{2} \longrightarrow 2$

$\frac{\sqrt{2}}{1}$

$\left(\sqrt{2}\right)^2 \longrightarrow$

$\sqrt{2}$

$\left((2)^{\frac{1}{2}}\right)^{2} = \boxed{2}$

Ex 2)

Convert the following:-

$$\sqrt{x} \longrightarrow x$$

$$\downarrow$$

$$\left(x^{\frac{1}{2}}\right)^{2} = x$$

$$\sqrt{(x+3)} \longrightarrow (x+3)$$

$$\downarrow$$

$$\left((x+3)^{\frac{1}{2}}\right)^{2} = (x+3)$$

$$\sqrt[5]{(x+3)^{2}} \longrightarrow (x+3)$$

$$\left((x+2)^{2}\right)^{\frac{1}{3}} = \left((x+2)^{\cancel{\frac{2}{3}}}\right)^{\boxed{\frac{3}{2}}}$$

$$\downarrow$$

$$(x+2)^{1}$$

$$\left(\boxed{}\right)^{3/2}$$

Ex) Solve: $\boxed{\sqrt{x+6} = x}$

<u>Step 1</u>: Get rid of roots

$$\left(\sqrt{x+6}\right)^2 = (x)^2$$

$$\downarrow$$

$$x+6 = x^2$$

$$x^2 - x - 6 = 0$$

$\sqrt[5]{\quad} \neq 5\sqrt{\quad}$

5th root $\qquad 5 \cdot \sqrt{\quad}$ $\boxed{(x+3)}$

$$\sqrt[5]{\boxed{(x+3)^2}} = \left((x+3)^2\right)^{\frac{1}{5}} = \boxed{(x+3)^{\frac{2}{5}}}$$

$$\left((x+3)^{\frac{5}{2}}\right)^{\frac{2}{5}}$$

$$\boxed{(x+3)}$$

$\sqrt{2}$

$$\left((2)^{\frac{1}{2}}\right)^{2} = 2^1$$

$$\sqrt[3]{(x+2)^2} \longrightarrow (x+2)$$

$x^2 + \underline{6x} + 9 = 0$

$x \quad + \quad 3 \longrightarrow (x+3)$

$x \quad + \quad 3 \longrightarrow (x+3)$

$\oplus 3x \quad + \quad \oplus 3x = 6x$ ✓

$(x+3)(x+3) = 0$

$\boxed{x = -3}$ or $\boxed{x = -3}$

Ex) $\sqrt{x+6} = x$

Step 1: square both sides

$ax^2+bx+c=$

$\left(\sqrt{(x+6)}\right)^2 = (x)^2$

$(x+6) = x^2$

$(x+2)(x-3)=0$

$\boxed{x=-2}$ or $\boxed{x=3}$ ✓

Step 2: make it QE

$x^2 \boxed{-x} - 6 = 0$

-1

$a=1 \quad b=-1 \quad c=-6$

QE Formula

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Step 3: Use $\boxed{x\text{-method}}$

$x \quad +2 \longrightarrow (x+2)$

$x \quad -3 \longrightarrow (x-3)$

$-3x \ldots = -x$

* Radical Equations

$|2x+1|=5$

$\sqrt{x}$ $\sqrt{2} \Rightarrow 2$ $\sqrt[3]{x}$

abe

$\sqrt{5x-3} + x = 1$ $\sqrt[4]{x}$ $|2x+1|=5$

$2x+1$

Examples

$\frac{\boxed{5}\sqrt{x+6}}{5} = \frac{x}{5}$ $\sqrt{x+1} = 1$

$\sqrt{x+1} = 5$, $\boxed{\sqrt{2x+1} = 5}$, $\sqrt{x+7} = x$

Ex1 Solve the following:- $\boxed{x = ?!}$

$\sqrt{x+6} = x$

Solution: $\sqrt{x+6} = x$

1

$$\boxed{x^2} - x - 6 = 0$$

Let's Kaabar x-method

Step 1: $\boxed{x^2}$

Step 2: -6

$x \quad + \quad 2 \longrightarrow (x+2)(x-3)=0$

$x \quad - \quad 3$

$+\,2x + (-)\,3x = -x$

$$\boxed{\begin{array}{c} x = -2 \\ \text{or} \\ x = 3 \end{array}}$$