Mohammed Kaabar Differentiation Theorems II March 4^{th}, 2016

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* Important Definitions:

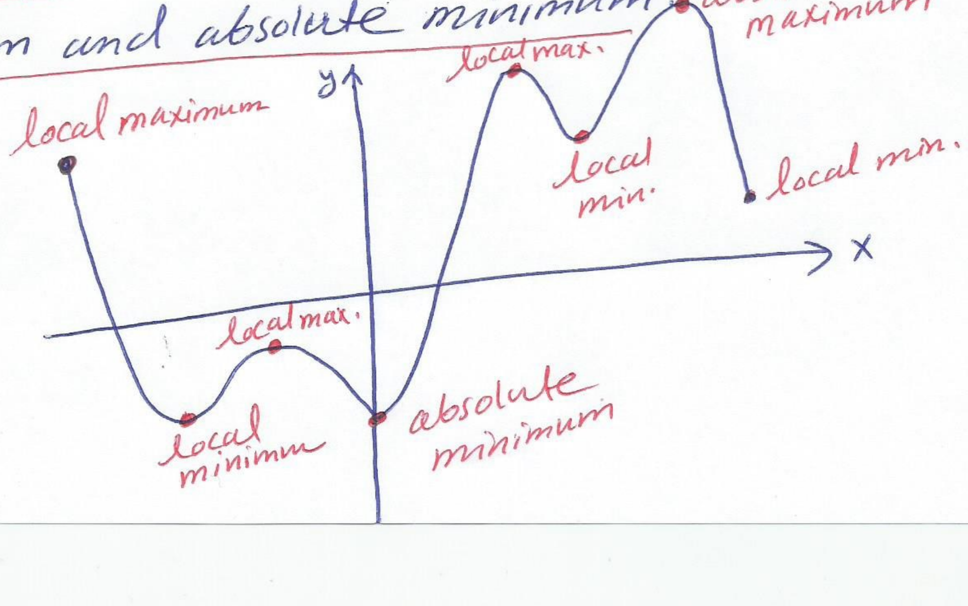
$\boxed{1-}$ f has an absolute (global) maximum at $x=c$ if $f(c) \geq f(x)$ for all x in D (domain).

$\boxed{2-}$ f has an absolute (global) minimum at $x=c$ if $f(c) \leq f(x)$ for all x in D (domain).

$\boxed{3-}$ f has a local (relative) maximum at $x=c$ if $f(c) \geq f(x)$ for x near c.

$\boxed{4-}$ f has a local (relative) minimum at $x=c$ if $f(c) \leq f(x)$ for x near c.

* Extreme Value Theorem: If f is a continuous on a closed interval $[a,b]$, then f assumes (to be) an absolute maximum and absolute minimum

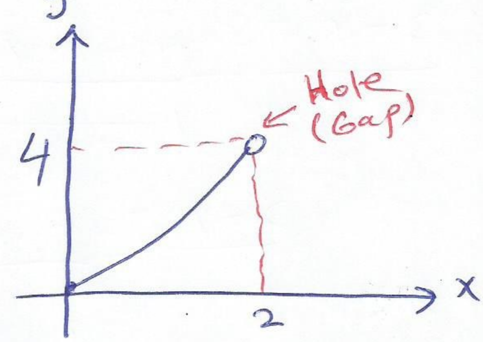


Ex1] Find and specify the max. & min. for $f(x) = x^2$ on $[0, 2)$.

Solution:

Absolute minimum $= 0$ at $x = 0$.

There is no absolute maximum.

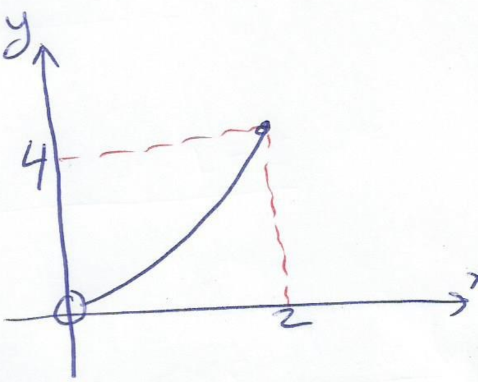


Ex2] Find and specify the max. & min. for $f(x) = x^2$ on $(0, 2]$.

Solution:

There is no absolute minimum.

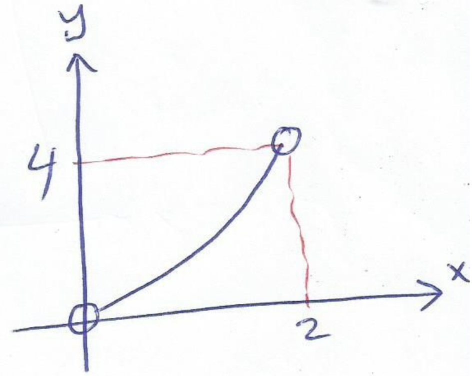
Absolute maximum $= 4$ at $x = 2$



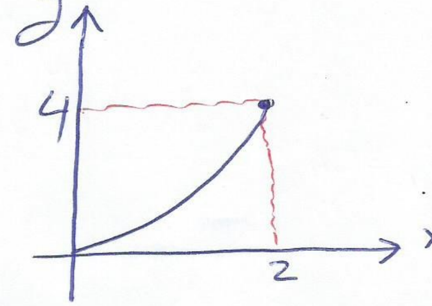
Ex3] Find and specify the max. & min. for $f(x) = x^2$ on $(0, 2)$.

Solution:

There are no absolute maximum and absolute minimum.



Ex4) Find and specify the max. & min. for $f(x) = x^2$ on $[0,2]$.



Solution:

Absolute maximum $= 4$ at $x = 2$

Absolute minimum $= 0$ at $x = 0$

Ex5) Find and specify the max. & min. for the following given graph:

Solution:

Absolute minimum $= 1$ at $x = 5$.

There is no absolute maximum.

* Three things you need to know to find the absolute extreme points.

$f(x)$ is continuous on $[a,b]$

1- Endpoints $x = a$ and $x = b$.

2- $f'(x) = 0$

3- $f'(x)$ does not exist.

Ex6 Find the abs. extreme points for $f(x) = x^2$ on $[-2,1]$.

Solution:

f is continuous (polynomial) on a closed interval so by Extreme Value Theorem, f has an abs. max. and abs. min.

1- $\boxed{x=-2}$ and $\boxed{x=1}$ (Endpoints)

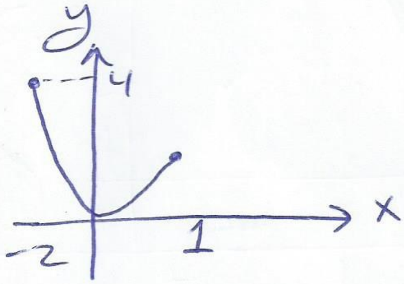
2- $f'(x)=0 \Rightarrow f'(x) = 2x = 0 \Rightarrow \boxed{x=0}$

3- f' always exists

$-2 \longrightarrow 4$ abs. max

$1 \longrightarrow 1$

$0 \longrightarrow 0$ abs. min.



Ex7 Find the abs. extreme points for $f(x) = \underbrace{x^{2/3}}_{\sqrt[3]{x^2}}$ on $[-2,1]$.

Solution:

By using Extreme Value Theorem, we do the following:

1- Endpoints: $\boxed{x=-2}$ and $\boxed{x=1}$.

2- $f'(x)=0 \Rightarrow f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} \neq 0$

3- $f'(x)$ does not exist at $\boxed{x=0}$.

$-2 \longrightarrow \sqrt[3]{4}$ abs. max.

$1 \longrightarrow 1$

$0 \longrightarrow 0$ abs. min.