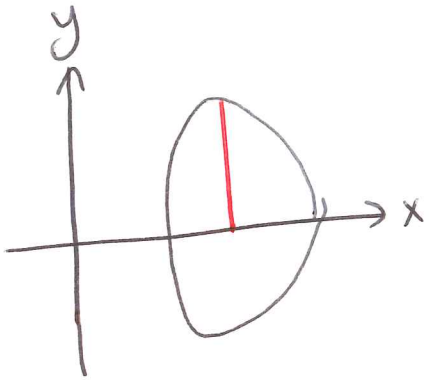
Mohammed Kaabar

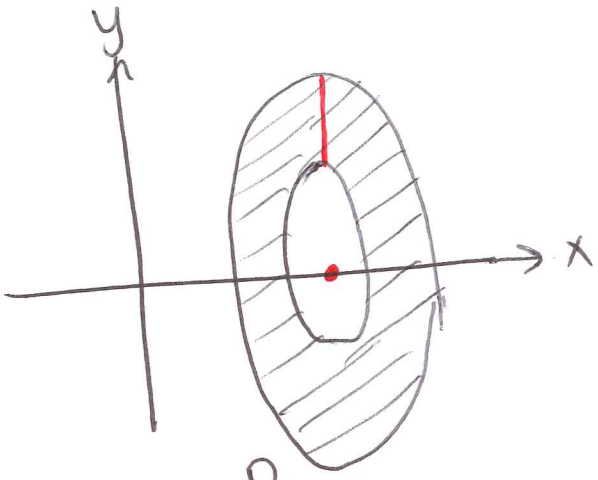
Volumes by Slicing and Shells

Sept. 8th, 2015

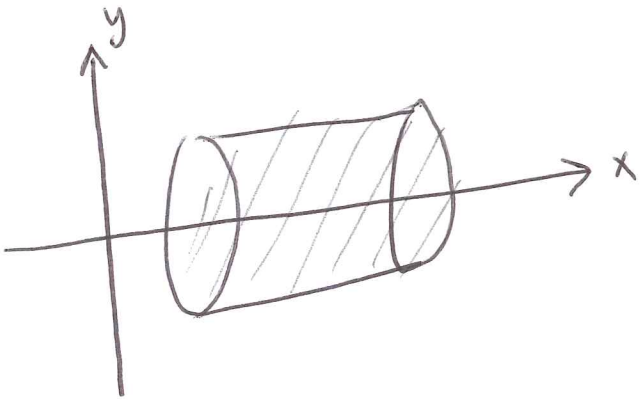
Volumes:



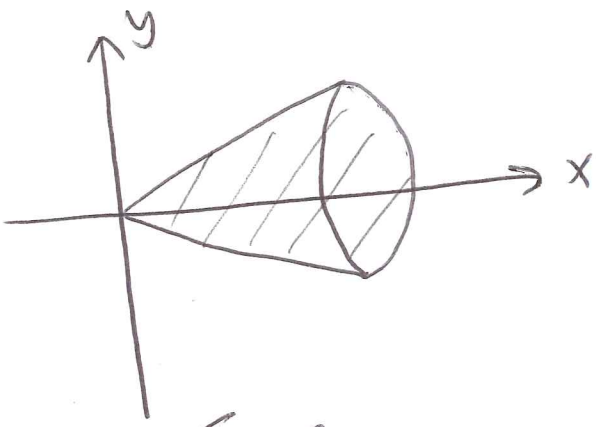
Circle



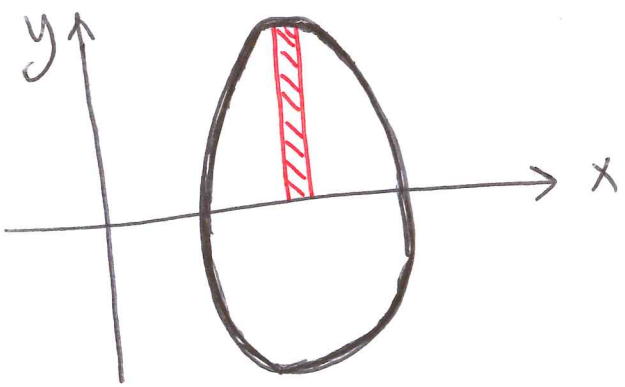
Ring



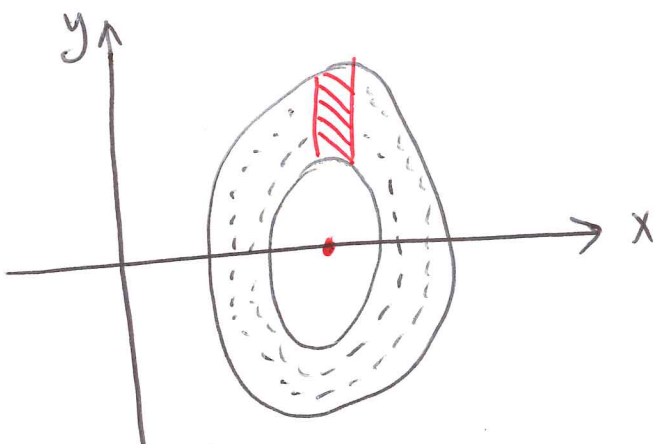
Cylinder



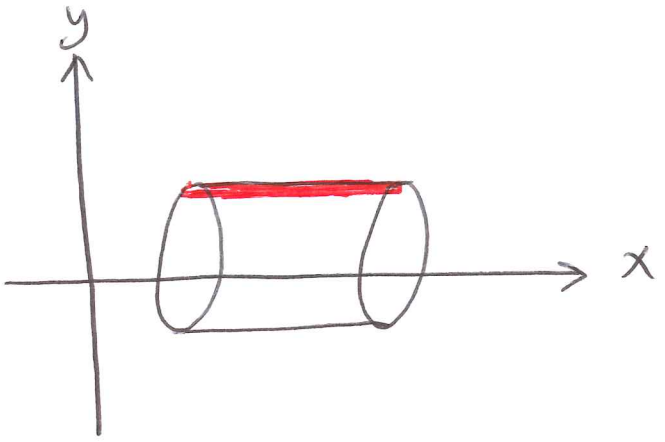
Cone



Disk



Washer

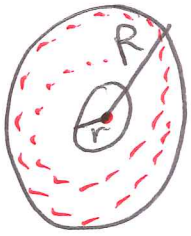


Shell

Volumes: ① Volume of Disk

$$V = \pi r^2 t$$

r: radius, t: thickness



② Volume of Washer

$$V = (\pi R^2 - \pi r^2) t$$

③ Volume of Shell

$$V = 2\pi r h t$$

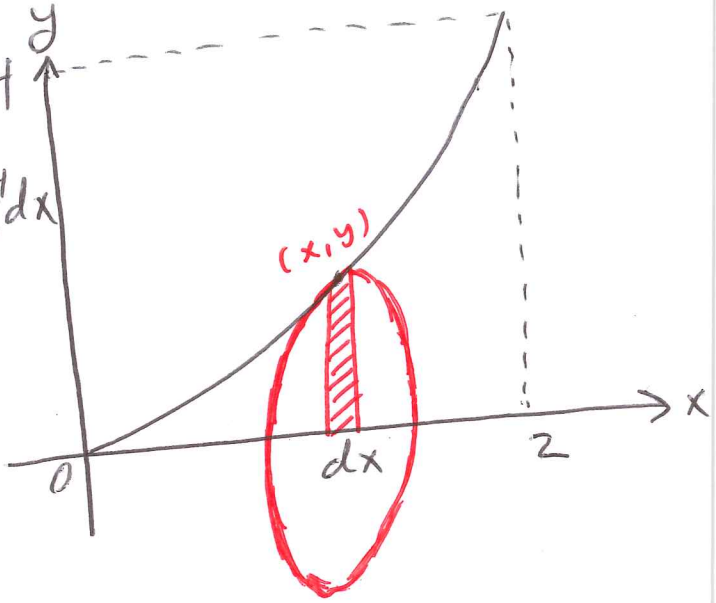
h: height

Example: Find the volume of the solid generated by revolving the region bounded by $y = x^2$ and the x-axis from $x = 0$ to $x = 2$

Part (a): about x-axis

$$V = \int_0^2 \pi y^2 \, dx = \pi \int_0^2 (x^2)^2 \, dx = \pi \int_0^2 x^4 \, dx$$

$$= \left[\frac{\pi (2)^5}{5} - \frac{\pi (0)^5}{5} \right] = \boxed{\frac{32\pi}{5}}$$



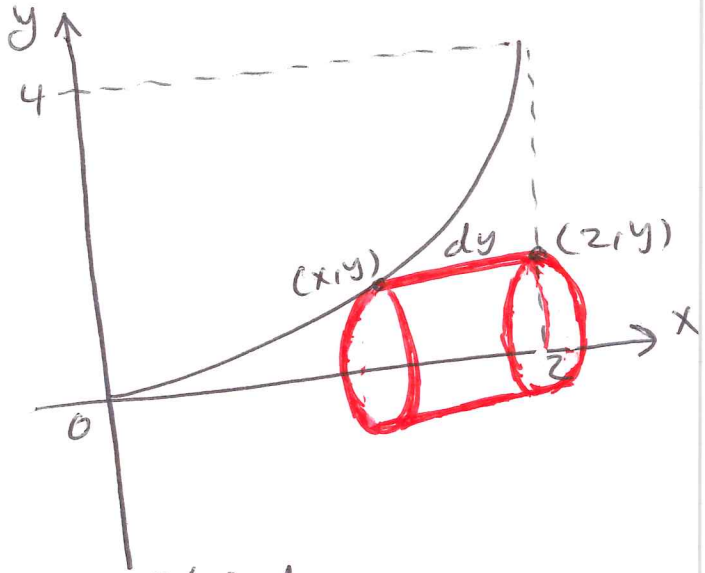
OR (Another Way)

$$V = \int_0^4 2\pi y (2 - x) \, dy$$

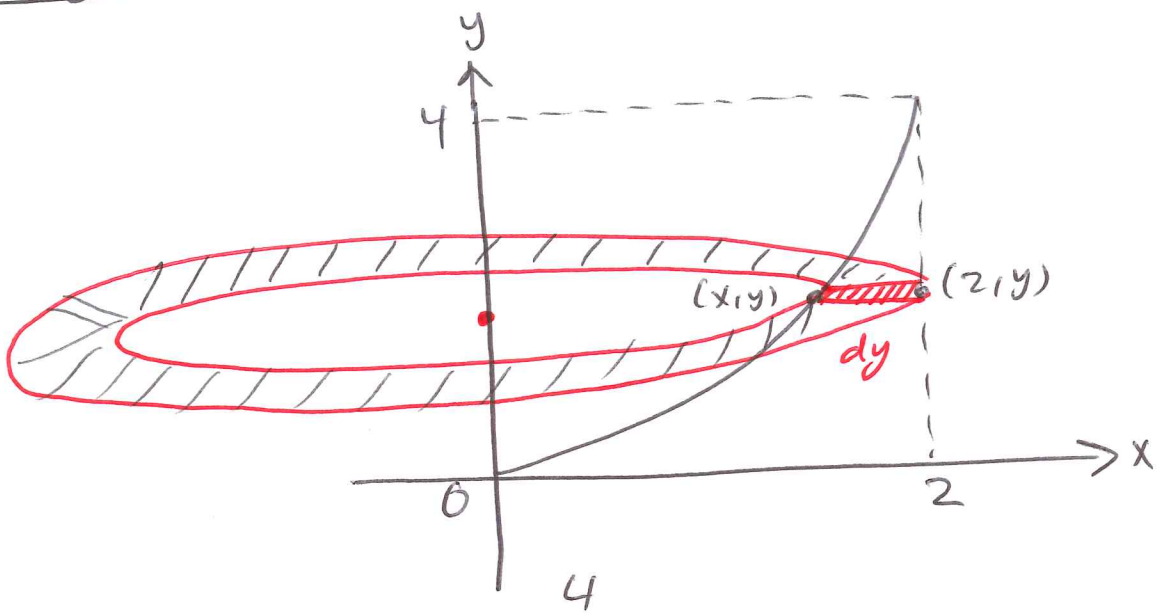
$$= \int_0^4 2\pi y (2 - y^{1/2}) \, dy$$

$$= \int_0^4 2\pi \left[2y - y^{3/2} \right] dy = 2\pi \int_0^4 (2y - y^{3/2}) \, dy$$

$$= 2\pi \left[y^2 - \frac{y^{5/2}}{5/2} \right] \Bigg|_0^4 = \boxed{\frac{32\pi}{5}}$$



Part (b): about y-axis



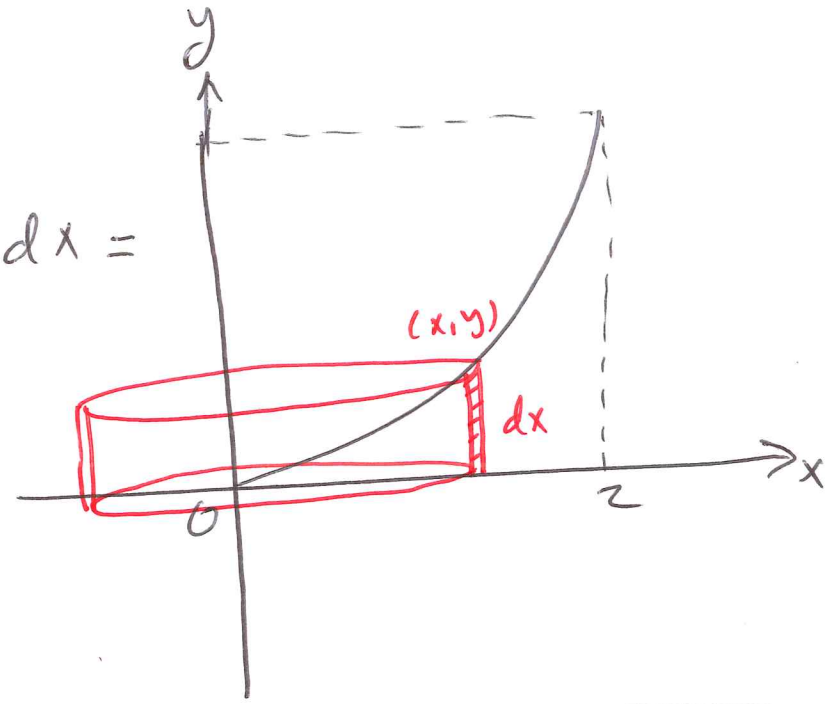
$$V=\int_0^4 [\pi(2)^2-\pi(x)^2]\,dy = \pi\int_0^4 [4-y]\,dy =$$

$$= \pi\left(4y-\frac{y^2}{2}\right)\Big|_0^4 = \pi\left[16-8\right] = \boxed{8\pi}$$

OR (Another Way)

$$V=\int_0^2 2\pi x y\,dx = 2\pi\int_0^2 x^3\,dx =$$

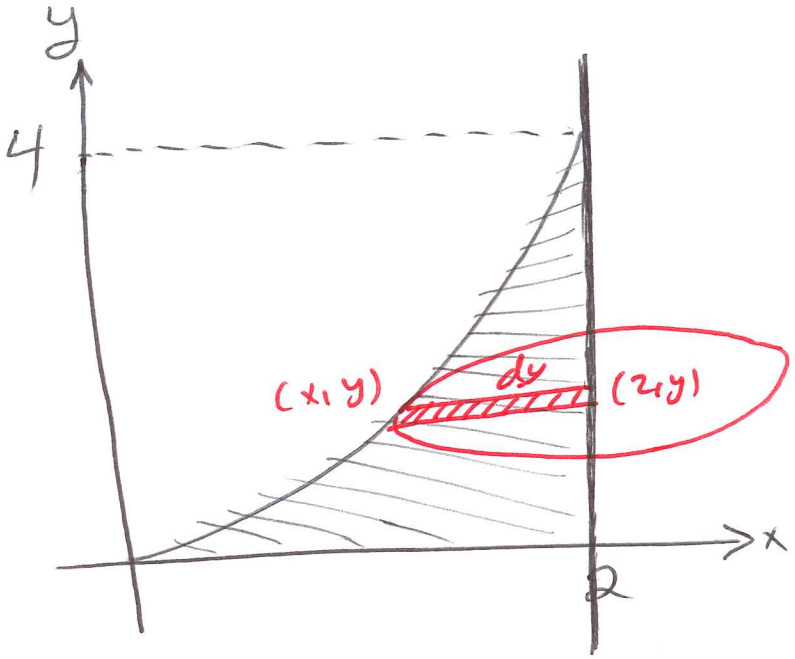
$$= 2\pi\,\frac{x^4}{4}\Big|_0^2 = \boxed{8\pi}$$



Part (C): About $x = 2$

$$V = \int_0^4 \pi (2 - x)^2 \, dy$$

$$= \pi \int_0^4 (2 - \sqrt{y})^2 \, dy$$



OR (Another Way)

$$V = \int_0^2 2\pi (2 - x) y \, dx$$

$$= 2\pi \int_0^2 (2 - x) x^2 \, dx$$