Handout 5

MATH 172 Lab: Sections 7 and 8

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Note: This handout covers only differential equations and integration by parts.

Instruction: Work in groups to solve the following mathematical problems. DON'T AFRAID TO MAKE MISTAKES BECAUSE WE LEARN FROM OUR MISTAKES!

Problem 1: Find the general solution of the given differential equation:

$$\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

(**Hint:** General solution means that you need to write it as $y(x)$ as we did in the Differential Equations Lab on Thursday)

$$\frac{dy}{dx}\times\frac{\frac{1}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-y^2}}}\Rightarrow\frac{1}{\sqrt{1-y^2}}dy=\frac{1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow\frac{1}{\sqrt{1-y^2}}dy-\frac{1}{\sqrt{1-x^2}}dx=0$$

$$\Rightarrow\int\frac{1}{\sqrt{1-y^2}}dy-\int\frac{1}{\sqrt{1-x^2}}dx=\int 0$$

$$\Rightarrow \sin^{-1}(y)-\sin^{-1}(x)=C$$

$$\Rightarrow \sin^{-1}(y)=\sin^{-1}(x)+C$$

Take $\sin$ of both sides $\Rightarrow \sin\left(\sin^{-1}(y)\right)=\sin\left(\sin^{-1}(x)+C\right)$

$$\Rightarrow \boxed{y(x)=\sin\left(\sin^{-1}(x)+C\right)=x+C}$$

where C is a constant

Problem 2: Find the general solution of the given differential equation:

$$\frac{dy}{dx} = 3xe^{(x+5y)} \longrightarrow e^x \cdot e^{5y}$$

(**Hint:** General solution means that you need to write it as $y(x)$ as we did in the Differential Equations Lab on Thursday)

$$\Rightarrow \frac{dy}{dx} = 3xe^x \cdot e^{5y} \Rightarrow \frac{dy}{dx} \times \frac{3xe^x}{e^{-5y}}$$

$$\Rightarrow e^{-5y}\,dy = 3xe^x\,dx \Rightarrow e^{-5y}\,dy - 3xe^x\,dx = 0$$

$$\Rightarrow \int e^{-5y}\,dy - 3\int xe^x\,dx = C$$

Table Method

derivative	Integration
x	e^x
1	e^x (+)
0	e^x (−)

$$\Rightarrow \frac{e^{-5y}}{-5} - 3[xe^x - e^x] = C$$

$$\Rightarrow -\frac{1}{5}e^{5y} - 3xe^x + 3e^x = C$$

Problem 3: Evaluate the following integral:

$$\int \tan^{-1}(x)\,dx$$

By parts:

$u = \tan^{-1}(x)$ $\quad dv = dx$

$du = \frac{1}{1+x^2}dx$ $\quad v = x$

$$\int \tan^{-1}(x)dx = x\tan^{-1}(x) - \int \frac{x}{1+x^2}dx$$

$$\int \tan^{-1}(x)dx = x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C$$

$$\int \frac{x}{1+x^2}dx \qquad w = 1+x^2$$

$dw = 2(x\,dx) \Rightarrow x\,dx = \frac{dw}{2}$

$$\frac{1}{2}\int \frac{1}{w}dw = \frac{1}{2}\ln|w| + C = \frac{1}{2}\ln|1+x^2| + C$$

$$-\frac{1}{5}e^{5y} = 3xe^x - 3e^x + C$$

multiply (-5) both sides

$$e^{5y} = -15xe^x + 15e^x + C$$

Take ln of both sides

$$\ln e^{5y} = -15x\ln e^x + 15\ln e^x + C$$

$$5y = -15x \cdot x + 15x + C$$

$$5y = -15x^2 + 15x + C$$

Divide everything by 5,

$$y(x) = -3x^2 + 3x + C$$

where C is a constant.

Challenging Problem: Solve the following differential equation:

$$\frac{dy}{dx} = \frac{\sin(5x+y)}{\cos(5x+y) - 2\sin(5x+y)} - 5$$

(**Hint:** No need to write your solution as $y(x)$)

Let $w = 5x + y$

$$\frac{dw}{dx} = 5 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{dw}{dx} - 5$$

$$\Rightarrow \frac{dw}{dx} - 5 = \frac{\sin(5x+y)}{\cos(5x+y) - 2\sin(5x+y)} - 5$$

$$\Rightarrow \frac{dw}{dx} = \frac{\sin(w)}{\cos(w) - 2\sin(w)}$$

$$\Rightarrow \frac{dw}{dx} = \frac{1}{\frac{\cos(w) - 2\sin(w)}{\sin(w)}}$$

$$\Rightarrow \frac{\cos(w) - 2\sin(w)}{\sin(w)}\, dw = dx \Rightarrow \frac{\cos(w) - 2\sin(w)}{\sin(w)}\, dw - dx = 0$$

$$\Rightarrow \int \frac{\cos(w) - 2\sin(w)}{\sin(w)}\, dw - \int dx = \int 0$$

$$\Rightarrow \int \frac{\cos(w)}{\sin(w)}\, dw - 2\int \frac{\sin(w)}{\sin(w)}\, dw - x = C$$

$$\Rightarrow \ln|\sin(w)| - 2w - x = C$$

Good Luck in Quiz 3

Best Regards,

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$$\Rightarrow \ln|\sin(5x+y)| + 2(5x+y) - x = C$$

$$\Rightarrow \boxed{\ln|\sin(5x+y)| + (10x + 2y) - x = C}$$ where C is a constant.