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Differential Equations & Integration by Parts

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Definition 1: A mathematical equation is called differential equation if it has 2 types of variables: dependent and independent variables where the dependent variable can written in terms of independent variable.

Example 1: Given: $y' = 15x$.

a. Find y.

b. Determine whether $y' = 15x$ is a linear differential equation or non-linear differential equation.

c. Find the order of this differential equation.

Solution: Part (a): To find y, we do the following:

$$\int y' \, dy = \int 15x \, dx$$

$$y = \int 15x \, dx = 15 \frac{x^2}{2} + C = \boxed{7.5x^2 + C}$$ where C is a constant. So, y is called dependent variable because it depends on x while x is called independent variable because it does not depend on y.

$\Longrightarrow$

Part (b):

Definition 2: The differential equation is called $\boxed{\text{linear}}$ if the dependent variable and all its derivatives are to the power 1. Otherwise, the differential equation is $\boxed{\text{non-linear}}$

Now, Since $y(x) = 7.5x^2 + C$ where c is a constant, then

dependent variable (y) — independent variable (x)

we can see that the dependent variables and all its derivatives are to the power 1. Thus, by definition 2, this differential equation is <u>linear</u>.

Part (c):

Definition 3: The order of differential equation is the highest derivative in the equation, i.e. the order of this equation: $y^{(3)} + 3y^{(2)} + 2y' = 12x^2 + 22$ is $\underline{3}$.

Hence, by definition 3, our differential equation $y' = 15x$ has an order of 1.

Example 2: Given: $m^{(4)} + (3m^{(2)})^3 - m = \sqrt{x+1}$

a. Is it linear differential equation? Why?!

b. What is the order of this differential equation?

$\Longrightarrow$

Part (a):

$$\underbrace{m^{(4)}}_{\text{dependent variable}} + \underbrace{(3m^{(2)})^3}_{\text{are not to the power 1.}} - m = \underbrace{\sqrt{x+1}}_{\text{independent variable}}$$

By applying definition 2, we see that the dependent variable, m, and all its derivatives are NOT to the power 1. Hence, it's non-linear differential equation.

Part (b):
By applying definition 3, the highest derivative is 4. So, the order of this differential equation is 4.

Note: For the $\boxed{\text{Separable Method}}$ in differential equation, please look at Handout #4 or my textbook: (A Friendly Introduction to Differential Equations).

Integration by Parts:

$(uv)' = uv' + u'v$

$uv' = (uv)' - u'v$

let's integrate both sides with respect to x.

$\int uv'dx = \int (uv)'dx - \int u'vdx$

$\int uv'dx = uv - \int u'v dx \rightarrow \int \boxed{v\frac{du}{dx}} dx$

$$\boxed{\begin{aligned}\int uv'dx &= \int u\frac{dv}{dx}dx \\ &= \int u\,dv \\ \int u'vdx &= \int v\frac{du}{dx}dx \\ &= \int v\,du\end{aligned}}$$

$\Longrightarrow$