Mohammed Kaabar | Sample Exam 1 Math 140 Dr. Elissa Exam 1 (Sample Exam) | Feb. 18, 2016

Question 1: $y = \dfrac{\ln x}{e^x}$

Solution: $y' = ?$ To find the derivative, you have two options:

Option 1: Re-write: $y = \dfrac{\ln x}{e^x}$ as $y = (\ln x)(e^{-x})$

Then, use product rule to find y' as follows:

$$y' = \left(\frac{1}{x}\right)(e^{-x}) + (-e^{-x})(\ln x)$$

$$y' = e^{-x}\left(\frac{1}{x} - \ln x\right) = e^{-x}\left(\frac{1 - x\ln x}{x}\right) = \boxed{\frac{1}{e^x}\left(\frac{1 - x\ln x}{x}\right)}$$

Option 2: Use quotient rule to find y' as follows:

$$y' = \frac{(e^x)\left(\frac{1}{x}\right) - (\ln x)(e^x)}{(e^x)^2} = \frac{\cancel{e^x}\left(\frac{1}{x} - \ln x\right)}{(\cancel{e^x})(e^x)} =$$

because $(e^x)^2 = (e^x)\cdot(e^x)$

$$= \frac{\left(\frac{1}{x} - \ln x\right)}{e^x} = \frac{\left(\frac{1 - x\ln x}{x}\right)}{e^x} = \left(\frac{1 - x\ln x}{x}\right)\cdot\left(\frac{1}{e^x}\right) =$$

$$= \boxed{\frac{1}{e^x}\left(\frac{1 - x\ln x}{x}\right)}.$$

Question 2: $y = \ln(5x+3)$

Solution: Use: $(\ln \square)' = \frac{1}{\square} \cdot \square'$

$(5x+3)$ derivative of $(5x+3)$ is 5

Then, $y' = \frac{1}{(5x+3)} \cdot (5)$

$$\boxed{y' = \frac{5}{(5x+3)}}$$

Question 3: $y = (8x^4 - 3x^2)^3$

Solution: Use $(\square)^{\Delta} = \Delta \cdot (\square)^{\Delta - 1} \cdot (\square')$

$$\boxed{y' = 3(8x^4 - 3x^2)^2 \cdot (32x^3 - 6x)}$$

Question 4: $f(x) = 3xe^{2x}$

Solution: $f(x) = (3x)(e^{2x})$, then use product rule as follows:

$f'(x) = (3)(e^{2x}) + (3x)(2e^{2x})$

$f'(x) = e^{2x}(3 + 3(2)x) = \boxed{e^{2x}(3+6x)}$

Question 5: $f(x)=\sqrt{(2x+9)^3}$ at $x=0$.

Solution: Rewrite $f(x)=\sqrt{(2x+9)^3}$ as $f(x)=(2x+9)^{3/2}$

$$\sqrt{\square}=\square^{1/2}$$

$$\sqrt{\square^3}=\left(\square^3\right)^{1/2}=\square^{3/2}$$

$$f'(x)=\frac{3}{\cancel{2}}(2x+9)^{1/2}\cdot(\cancel{2})$$

$$f'(x)=3(2x+9)=3\left(\sqrt{2x+9}\right)$$

Now, find $f'(0)$ as follows:

$$f'(0)=3\left(\sqrt{2(0)+9}\right)=3\left(\sqrt{0+9}\right)=3(\sqrt{9})=3(3)=\boxed{9}$$

Question 6: $f(x)=\frac{6}{x}$ using the definition of derivative

Focus on this sentence (ONLY DEFINITION Accepted here) Be-careful!!!

Solution:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\left[\frac{\left(\frac{6}{x+h}\right)-\left(\frac{6}{x}\right)}{h}\right]$$

$$=\lim_{h\to 0}\left[\frac{\frac{6x-6(x+h)}{x(x+h)}}{h}\right]=\lim_{h\to 0}\left[\frac{\frac{\cancel{6x}-\cancel{6x}-6h}{x(x+h)}}{\frac{h}{1}}\right]=$$

$$=\lim_{h\to 0}\left[\frac{-6\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}}\right]=\lim_{h\to 0}\left(\frac{-6}{x(x+h)}\right)=\frac{-6}{x(x+0)}=\boxed{\frac{-6}{x^2}}$$

Question 7: $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 - 3x + 2}$.

Solution: $\lim\limits_{x \to 1} \dfrac{(x^2 - 1)}{(x^2 - 3x + 2)} \Rightarrow \dfrac{1 - 1}{1 - 3 + 2} = \boxed{\dfrac{0}{0}}$ Indeterminate

So, $\lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-2)} =$

$= \lim\limits_{x \to 1} \dfrac{(x+1)}{(x-2)} = \dfrac{1+1}{1-2} = \dfrac{2}{-1} = \boxed{-2}$

$(x^2 - 1) = (x-1)(x+1)$

How to Simplify $(x^2 - 3x + 2)$?

Answer:

$x \quad -1$

$x \quad -2$

$-x \quad -2x$

$\boxed{-3x}$

Question 8: $\lim\limits_{h \to 0} \dfrac{(x+h)^3 - x^3}{h}$

Solution: $\dfrac{(x+0)^3 - (x)^3}{0} = \dfrac{x^3 - x^3}{0} = \boxed{\dfrac{0}{0}}$ Indeterminate

$\lim\limits_{h \to 0} \dfrac{(x+h)^3 - x^3}{h} =$

$= \lim\limits_{h \to 0} \dfrac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$

$= \lim\limits_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2 + 3x\overset{0}{(0)} + \cancel{0}^{2^0} = \boxed{3x^2}$

Use Pascal's Triangle to find $(x+h)^3 = ??$

1 $n = 0$

1 1 $n = 1$

1 2 1 $n = 2$

$\boxed{1 \quad 3 \quad 3 \quad 1}$ $n = 3$

$(x+h)^3 = 1 \cdot x^3 + 3x^2h + 3xh^2 + 1 \cdot h^3$

$= \boxed{x^3 + 3x^2h + 3xh^2 + h^3}$

Question 9: $\lim_{x\to\infty} \frac{5x^3+9x-6}{4x^4+5x^3+4}$.

Solution: $\frac{5(\infty)^3+9(\infty)-6}{4(\infty)^4+5(\infty)^3+4} = \boxed{\frac{\infty}{\infty}}$ Indeterminate

Now, we use the leading terms method

$$\lim_{x\to\infty} \frac{5x^3+9x-6}{4x^4+5x^3+4} = \lim_{x\to\infty} \frac{5\cancel{x^3}}{4x^{\cancel{4}}} = \lim_{x\to\infty} \frac{5}{4x} = \frac{5}{4(\infty)} = \boxed{0}$$

Question 10: $f(x) = x^3+6x^2+21x+2$

$m = 9$

Find $(x_1, y_1) = ?$ and $\boxed{y-y_1 = m(x-x_1)}$ Eq. of Tangent line

Solution:

We know $m = f'(x) \Rightarrow$ So, $9 = \boxed{f'(x)}$

$9 = \boxed{3x^2+12x+21} \Rightarrow$ So, $3x^2+12x+\boxed{21-9}=0$

$\Rightarrow 3x^2+12x+12=0$ Now, Solve: $3x^2+12x+12=0$

$\Rightarrow (x+2)(3x+6)=0$

$\Rightarrow \boxed{x=-2}$ or $3x+6=0 \Rightarrow \frac{3x}{3} = \frac{-6}{3}$ $\boxed{x=-2}$

$1 \cdot x$	$+2$
$3 \cdot x$	$+6$
$6x$	$+6x$
$12x$	

follow $\Longrightarrow$

So, $\boxed{x = -2}$ now let's find $y_1 = ?$ by plugging

$x = -2$ in $x^3 + 6x^2 + 21x + 2$ as follows:

$$f(-2) = (-2)^3 + 6(-2)^2 + 21(-2) + 2 = \boxed{-24 = y_1}$$

$$\text{So, } (x_1, y_1) = \boxed{(-2,-24)}$$

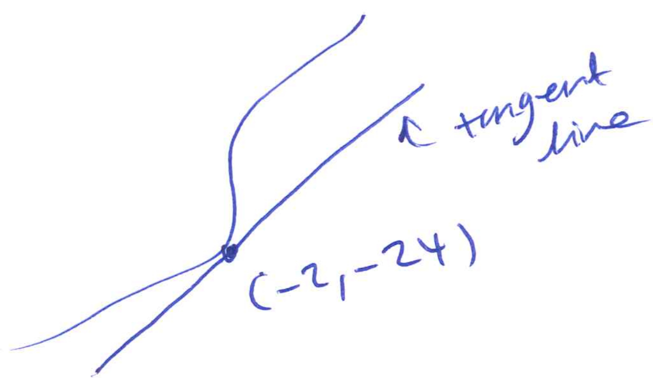
Now, let's find the equation of tangent line

as follows: $(\overset{x_1}{-2}, \overset{y_1}{-24})$

$$y - y_1 = m(x - x_1)$$

$$y + 24 = 9(x+2)$$

$$y = 9x + 18 - 24$$



$\boxed{y = 9x - 6}$ ← This is the equation of tangent line at $(-2,-24)$