Handout 6

MATH 172 Lab: Sections 7 and 8

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Student's ID:----------**Solution**-----

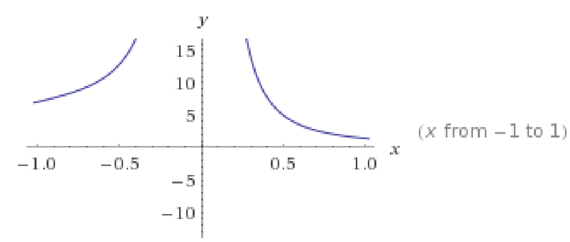
Note: *This handout covers only partial fractions and improper integrals.*

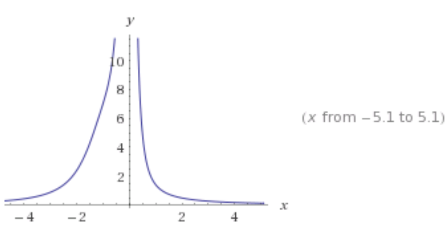
Instruction: Work in groups to solve the following mathematical problems. DON'T AFRAID TO MAKE MISTAKES BECAUSE WE LEARN FROM OUR MISTAKES!

Problem 1: Decompose $\frac{4x^2+3}{x^4+2x^3+2x^2}$ into partial fractions. Be sure to find the values of any unknown constants.

Solution:

$$\frac{4x^2+3}{x^4+2x^3+2x^2} = \frac{3x+11}{2(x^2+2x+2)} + \frac{3}{2x^2} - \frac{3}{2x}$$





Other Possible Different Forms for the above Problem:

$$\frac{4x^2+3}{x^2(x(x+2)+2)} \qquad \frac{4x^2+3}{x^2(x^2+2x+2)} \qquad \frac{8x^2+6}{2x^2(x^2+2x+2)}$$

Problem 2: Compute the following improper integral by evaluating appropriate limits:

$$\int_0^\infty \frac{1}{(1+y^2)(1+\tan^{-1} y)}\,dy$$

Solution:

$$\int_0^\infty \frac{1}{(1+y^2)(1+\tan^{-1} y)}\,dy = \lim_{t\to\infty}\int_0^t \frac{1}{(1+y^2)(1+\tan^{-1} y)}\,dy$$

Now, by substitution:

$u = (1+\tan^{-1} y)$

$du = \dfrac{1}{1+y^2}\,dy$

Hence, $\int \frac{1}{u}\,du = \ln|u| + c = \ln|(1+\tan^{-1} y)| + c$

Thus, $\lim_{t\to\infty}\int_0^t \frac{1}{(1+y^2)(1+\tan^{-1} y)}\,dy = \lim_{t\to\infty}[\ln|(1+\tan^{-1} t)| - \ln|(1+\tan^{-1}(0))|] =$
$\lim_{t\to\infty}[\ln|(1+\tan^{-1} t)| - \ln|(1+0|] = \lim_{t\to\infty}\left[\ln\left|1+\frac{\pi}{2}\right| - 0\right] = \ln\left|1+\frac{\pi}{2}\right|$

It is convergent.