Handout 3

Extra Credit

MATH 140 Lab: Section 1

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Note: *This handout covers the applications of exponential and logarithmic functions.*

Problem 1: Assume that $P(t)$ is the population of Pullman, WA after t years is given by the following model: $P(t) = 4000e^{0.034t}$. Suppose that $t = 0$ is the year 1980.

a. Find the initial population.

b. Find the population in the year 2010.

c. When will the population double to reach 8000?

(a) $P(0) = 4000e^{0.034(0)} = 4000e^{0} = 4000(1) = 4000$

So, the initial population is 4000.

(b) $t = 0 \longrightarrow$ the year 1980

$t = 30 \longrightarrow$ the year 2010

So, $P(30) = 4000e^{0.034(30)} = 11093$

thus, the population in the year 2010 is 11093.

(c) $P = 8000 \Longrightarrow \frac{8000}{4000} = \frac{4000e^{0.034t}}{4000} \Longrightarrow 2 = e^{0.034t} \Longrightarrow$

$\Longrightarrow \ln(2) = \ln e^{0.034t} \Longrightarrow \frac{\ln(2)}{0.034} = \frac{0.034t}{0.034} \Longrightarrow$

$\Longrightarrow t = \frac{\ln(2)}{0.034} \approx 23$. So, the population reaches 8000 after 23 years $\Longrightarrow$ in 2003.

Problem 2: Assume that M is the mass of a quantity of the radioactive radium-226. The amount of radium present after t years is given by: $M = 30(0.5)^{\frac{t}{1620}}$

a. Find the initial quantity of radium.
b. Find the quantity present after 500 years.
c. What is the half-life of radium-226?

Hint: The half-life of radioactive substance is the time required for half the mass to decay.

(a) $t=0 \Rightarrow M(0) = 30(0.5)^{\frac{0}{1620}}$

$\Rightarrow M(0) = 30(0.5)^0 = 30(1) = 30$

So, the initial quantity of radium is 30.

(b) $t=500 \Rightarrow M(500) = 30(0.5)^{\frac{500}{1620}} \approx 24.22$

So, the quantity present after 500 years is 24.22.

(c)

The initial amount is 30, so the half-time will be

$30\left(\frac{1}{2}\right) = \boxed{15}$

$M = 15 \Rightarrow \frac{15}{30} = \frac{30(0.5)^{\frac{t}{1620}}}{30} \Rightarrow 2 = (0.5)^{\frac{t}{1620}}$

$\Rightarrow \ln\left(\frac{1}{2}\right) = \ln(0.5)^{\frac{t}{1620}}$

$\Rightarrow \frac{\ln(0.5)}{\ln(0.5)} = \frac{t}{1620}\frac{\ln(0.5)}{\ln(0.5)}$

$\Rightarrow \frac{1}{1} = \frac{t}{1620} \Rightarrow \boxed{t = 1620}$

Thus, the half-time of radium-226 is $t = 1620$ years.