Study Guide 2

MATH 140 Lab: Section 1

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Note: *This study guide contains my practice questions that I think will be useful for preparing you for the second exam in Calculus for Life Scientists.*

Question 1: Find the derivative for the following:
Hint: Use Implicit Differentiation to find y'

$$e^y + xy^3 = 5x$$

$$e^y \cdot y' + (x)(3y^2 \cdot y') + (1)(y^3) = 5$$

$$e^y \cdot y' + (x)(3y^2 \cdot y') = 5 - y^3$$

$$y'(e^y + 3xy^2) = 5 - y^3 \Rightarrow \boxed{y' = \frac{5-y^3}{e^y + 3xy^2}}$$

Question 2: Find the equation of the tangent line at the point $(0, \pi)$ to the following curve:

x_1, y_1

$$x^2 \cos^2 y - \sin y = -x$$

$$2x \cdot \cos^2 y + x^2 \cdot 2(\cos y)(-\sin y) \cdot y' - \cos y \cdot y' = -1$$

$$y' = \frac{-1 - (2x\cos^2 y)}{[(x^2 \cdot 2\cos y) \cdot (-\sin y) - \cos y]}$$

$$y'\Big|_{(0,\pi)} = \frac{-1 - (2(0)\cos^2(\pi))}{[((0)^2 \cdot 2\cos(\pi))(-\sin(\pi)) - \cos(\pi)]} = \frac{-1}{-(-1)} = \boxed{-1}$$

$$y - y_1 = m(x - x_1) \Rightarrow y - \pi = -1(x - 0) \Rightarrow y - \pi = -x$$

$$\Rightarrow \boxed{y = \pi - x}$$

Question 3: Determine the values of x for which the function:

$$y = x^5 - 20x^2 + 1$$

is decreasing/increasing and determine concavity of the function. Find the location of maxima/minima and inflection points. Sketch the curve.

Solution

① y-intercept $(x=0) \Rightarrow \boxed{y=1}$

x-intercept $(y=0) \Rightarrow 0 = x^5 - 20x^2 + 1$

② $y' = 5x^4 - 40x$

$0 = 5x(x^3 - 8)$

$\boxed{x=0}$ & $x^3 - 8 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$\boxed{x=2}$

1st derivative Test

y': + + + (inc) | 0 (max.) | − − − (dec.) | 2 (min.) | + + + + (inc.)

③ $y'' = 20x^3 - 40$

$0 = 20(x^3 - 2)$

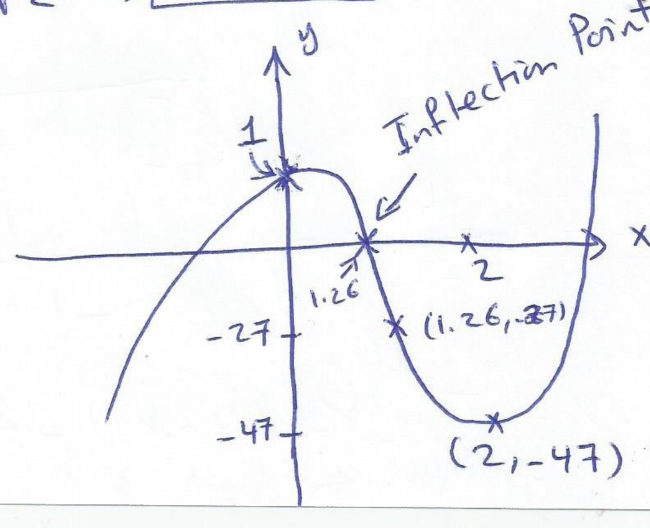
$20 \neq 0$

$\hookrightarrow x^3 - 2 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{2} \Rightarrow \boxed{x = \sqrt[3]{2}} \approx 1.26$

2nd derivative Test

y'': − − − − (concave down ∩) | $\sqrt[3]{2} \approx 1.26$ (inflection point) | + + + + (concave up ∪)



Question 4: Find the absolute extrema of the given function on $[-3,2]$.

$$f(x) = x^3 - 3x + 1$$

Solution:

$f(x)$ is continuous because it's polynomial function on a closed interval $[-3,2]$.

① Endpoint:

$\boxed{x=-3}$ & $\boxed{x=2}$

② $f'(x) = 3x^2 - 3 = 0$

$3x^2 = 3 \Rightarrow \boxed{x = \pm 1} \rightarrow \boxed{x=1}$ & $\boxed{x=-1}$

③ f' always exists

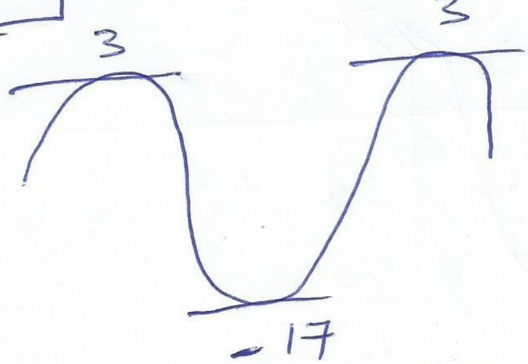
$-3 \longrightarrow -17 \longleftarrow$ Absolute Minimum

$2 \longrightarrow 3$

$1 \longrightarrow -1$

$-1 \longrightarrow 3$

Absolute Maximum



Question 5: Given a function:

$$f(x) = e^{-\frac{x^2}{2}}$$

Determine the intervals where the graph of f is concave up and concave down, then find the inflection points.

Solution:

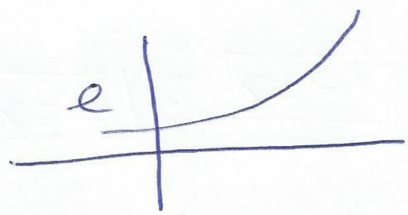
$$f'(x) = -x \cdot e^{-\frac{x^2}{2}}$$

$$f''(x) = x^2 \cdot e^{-\frac{x^2}{2}} + (-1) \cdot e^{-\frac{x^2}{2}} = x^2 e^{-\frac{x^2}{2}} - e^{-\frac{x^2}{2}}$$

$$f''(x) = 0 \Rightarrow x^2 e^{-\frac{x^2}{2}} - e^{-\frac{x^2}{2}} = 0 \Rightarrow e^{-\frac{x^2}{2}}(x^2 - 1) = 0$$

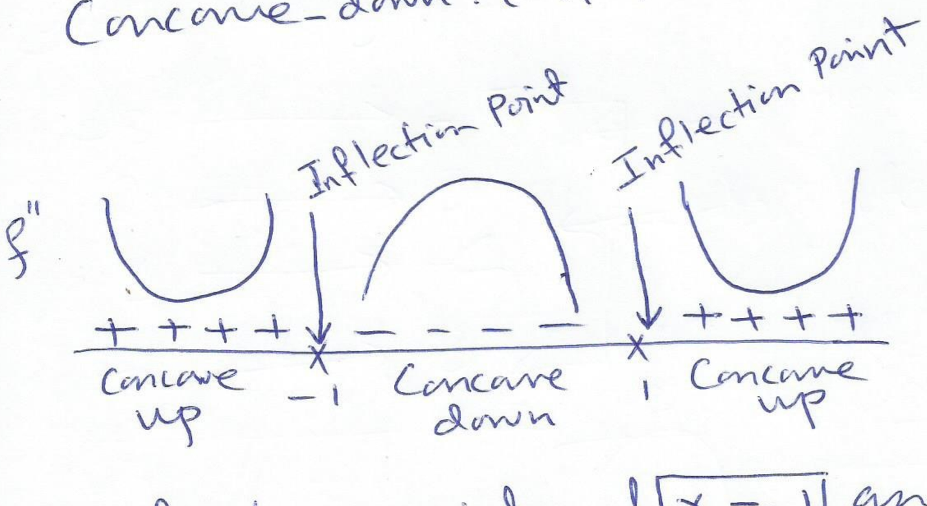
$$\Rightarrow e^{-\frac{x^2}{2}} \neq 0 \text{ and } \sqrt{x^2} = \sqrt{1} \Rightarrow \boxed{x = \pm 1}$$

The intervals of concavity:



Concave-up: $(-\infty, -1)$
$(1, \infty)$

Concave-down: $(-1, 1)$



Inflection points at $\boxed{x = -1}$ and $\boxed{x = 1}$.

Question 6: Find the integral for the following:

a. $\int \frac{4x}{x^2+3} dx$

b. $\int (3e^x - 2)dx$

c. $\int \frac{x^{\frac{1}{3}}-3}{x^{\frac{2}{3}}} dx$

d. $\int 2 \sec x \tan x \, dx$

(a) $\int \frac{4x}{x^2+3} dx = 2\int \frac{2x}{x^2+3} dx = \boxed{2\ln|x^2+3| + C}$

(b) $\int (3e^x - 2)dx = 3\int e^x dx - \int 2dx$

$= \boxed{3e^x - 2x + C}$

(c) $\int \frac{x^{1/3}-3}{x^{2/3}} dx = \int (x^{-1/3} - 3x^{-2/3})dx = \int x^{-1/3}dx - 3\int x^{-2/3}dx =$

$= \frac{x^{2/3}}{2/3} - 3\frac{x^{1/3}}{1/3} + C$

$= \boxed{\frac{3}{2}x^{2/3} - 9x^{1/3} + C}$

(d) $\int 2\sec x \tan x \, dx = \boxed{2\sec x + C}$

Question 7: Evaluate the integral:

$$\int x^{-3}(\sqrt[3]{x} - 3x^{-1} + 3)dx$$

$$= \int \left(x^{-\frac{8}{3}} - 3x^{-4} + 3x^{-3}\right)dx = \boxed{\frac{-3}{5}x^{-5/3} + x^{-3} - \frac{3}{2}x^{-2} + C}$$

Question 8: Evaluate the integral:

$$\int \frac{x^3}{\sqrt{1-x^4}}dx$$

By substitution:

let $u = 1 - x^4$

$\frac{du}{dx} = -4x^3$

$du = -4\boxed{x^3 dx}$

$\frac{du}{-4} = \boxed{x^3 dx}$

$$\int \frac{x^3}{\sqrt{1-x^4}}dx = \int \frac{1}{\sqrt{u}} \cdot \frac{du}{-4} = \frac{-1}{4}\int \frac{1}{\sqrt{u}}du = \frac{-1}{4}\int u^{-1/2}du =$$

$$= \frac{-1}{4}\frac{u^{1/2}}{1/2} = \frac{-1}{2}u^{1/2} = \frac{-1}{2}(1-x^4)^{1/2} = \boxed{\frac{-1}{2}\sqrt{1-x^4} + C}$$

Question 9: Find the following integral:

$$\int \left(\frac{8x+2}{x}\right)^2 dx$$

$$\int\left(\frac{8x+2}{x}\right)^2 dx = \int\left(\frac{64x^2+32x+4}{x^2}\right) dx = \int\left(\frac{64x^2}{x^2}+\frac{32x}{x^2}+\frac{4}{x^2}\right)dx$$

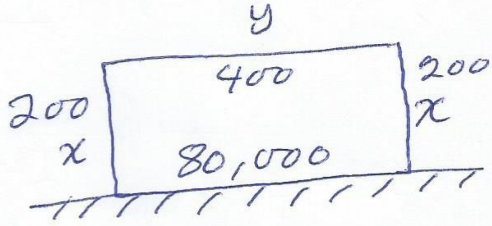
$$= \int\left(64+32x^{-1}+4x^{-2}\right)dx$$

$$= \boxed{64x+32\ln|x|-\frac{4}{x}+C}$$

Question 10: A rectangular plot of farmland will be bounded from one side by a river and from the other three sides by a fence. With a $\boxed{800\,ft}$ of the wire at your disposal. What is the largest area you can enclose?

Solution

$[0, 400]$



① $A = xy$

② $2x + y = 800$

$\Rightarrow \boxed{y = 800 - 2x}$

③ $A = x(800 - 2x)$

$A = 800x - 2x^2$

$A' = 800 - 4x = 0$

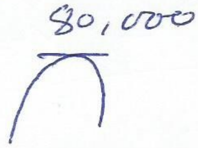
$\boxed{x = 200}$

$0 \longrightarrow 0$

$400 \longrightarrow 0$

$200 \longrightarrow 80{,}000$ ft absolute max.

$A'' = -4 < 0$



Good Luck in Exam 2

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