



Handout

MATH 172 Lab: Sections 7 and 8

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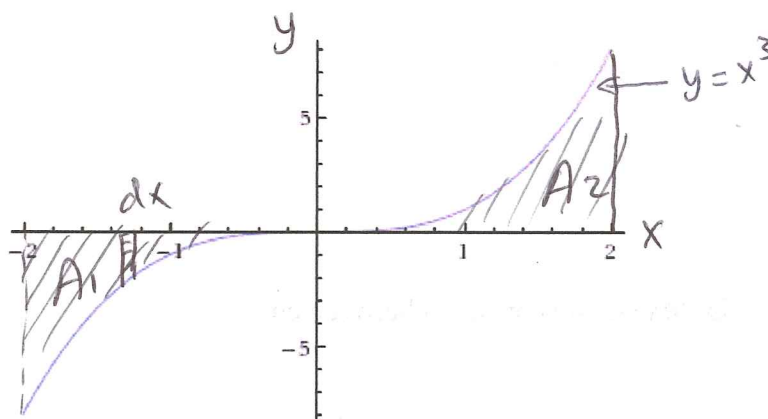
Student's Name: - Solution -

Student's ID: _____

Note: This handout covers some problems about the area between curves

Instruction: Work in groups to solve the following mathematical problems, and I want from each group one person to volunteer as a representative to present the solution of (one problem)/(one part of problem) on our class board. DON'T AFRAID TO MAKE MISTAKES BECAUSE WE LEARN FROM OUR MISTAKES!

Problem 1: The figure below shows the graph of $y = x^3$ and x - axis on the interval $[-2, 2]$.



Find the area between y and x - axis on $[-2, 2]$.

Hint: You can write the area as either one integral or a sum of two integrals (both answers are correct).

$$A = A_1 + A_2$$

$$= \int_{-2}^0 x^3 dx + \int_0^2 x^3 dx$$

$$= \left. \frac{x^4}{4} \right|_{-2}^0 + \left. \frac{x^4}{4} \right|_0^2$$

$$= (0 - 4) + (4 - 0)$$

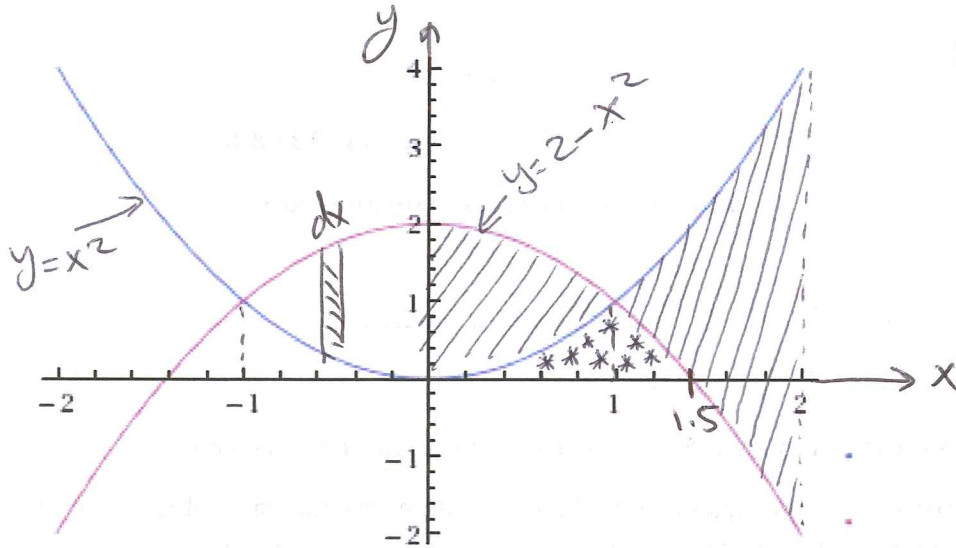
$$= (-4) + (4)$$

$$= 0$$

$$\text{or } \int_{-2}^2 x^3 dx = 0$$

which means that there is no area.

Problem 2: The figure below shows a region bounded by the functions $y = x^2$ and $y = 2 - x^2$.



a. Find the (highlighted area by lines) between curves.

$$A = \int_0^1 [(2-x^2) - (x^2)] dx + \int_1^2 [(x^2) - (2-x^2)] dx$$

$$= \frac{4}{3} + \frac{8}{3} = \boxed{4}$$

b. Find the (highlighted area by stars) between curves.

$$A = \int_0^1 [(x^2) - (0)] dx + \int_1^{1.5} [(2-x^2) - (0)] dx$$

$$= \frac{1}{3} + \frac{5}{24} = \boxed{\frac{13}{24}}$$

c. Find the area between two curves using intersection points.

$$x^2 = 2 - x^2$$

$$x^2 + x^2 = 2$$

$$2x^2 = 2$$

$$x^2 = 1 \Rightarrow \boxed{x=1} \text{ or } \boxed{x=-1}$$

$$A = \int_{-1}^1 [(2-x^2) - (x^2)] dx = \int_{-1}^1 (2 - 2x^2) dx =$$

$$= 2x - \frac{2x^3}{3} \Big|_{-1}^1$$

$$= \left(2(1) - \frac{2(1)^3}{3}\right) - \left(2(-1) - \frac{2(-1)^3}{3}\right) = \boxed{\frac{8}{3}}$$