

Problem Set 4 SOLUTIONS

Question 1: According to the following table of data for  $g(x)$ :

x	$g(x)$
2	3
3.5	1
7	2
10	0.25

Fit these data with linear spline interpolation.

SOLUTION using MAPLE 11

Linear Spline Interpolation

>  $g0 := x \rightarrow a0*x + b0;$

$$g0 := x \rightarrow a0 x + b0$$

>  $g1 := x \rightarrow a1*x + b1;$

$$g1 := x \rightarrow a1 x + b1$$

>  $g2 := x \rightarrow a2*x + b2;$

$$g2 := x \rightarrow a2 x + b2$$

>  $eq1 := g0(2) = 3;$

$$eq1 := 2 a0 + b0 = 3$$

>  $eq2 := g0(3.5) = 1;$

$$eq2 := 3.5 a0 + b0 = 1$$

>  $eq3 := g1(3.5) = 1;$

$$eq3 := 3.5 a1 + b1 = 1$$

>  $eq4 := g1(7) = 2;$

>

$$eq4 := 7 a1 + b1 = 2$$

>  $eq5 := g2(7) = 2;$

$$eq5 := 7 a2 + b2 = 2$$

>  $eq6 := g2(10) = 0.25;$

$$eq6 := 10 a2 + b2 = 0.25$$

```
> solve ({eq1, eq2, eq3, eq4, eq5, eq6});
      {b1 = 0., b0 = 5.666666667, a0 = -1.333333333, a2 = -0.5833333333,
      b2 = 6.083333333, a1 = 0.2857142857 }
```

Question 2: According to the following table of data for g(x):

x	g(x)
-2	0
0	3
1	1
3	2

Fit these data with quadratic spline interpolation.

**SOLUTION using MAPLE 11**

Quadratic Spline Interpolation:

```
> g0 := x-> a0*x^2 + b0*x + c0;
      g0 := x -> a0 x^2 + b0 x + c0

> g1 := x-> a1*x^2 + b1*x + c1;
      g1 := x -> a1 x^2 + b1 x + c1

> g2 := x-> a2*x^2 + b2*x + c2;
      g2 := x -> a2 x^2 + b2 x + c2
```

The 1st & 2nd functions equal at the interior knots

```
> eq1 := g0(0)=3;
      eq1 := c0 = 3

> eq2 := g1(0)=3;
      eq2 := c1 = 3
```

The 2nd & 3rd functions equal at the interior knots

```
> eq3 := g1(1)=1;
      eq3 := a1 + b1 + c1 = 1

> eq4 := g2(1)=1;
      eq4 := a2 + b2 + c2 = 1
```

The 1st & 3rd functions equal at endpoints

```
> eq5 := g0(-2)=0;
      eq5 := 4 a0 - 2 b0 + c0 = 0
```

> **eq6 := g2(3)=2;**

$$eq6 := 9 a2 + 3 b2 + c2 = 2$$

The 1st derivative equal at the interior knots

> **eq7 := D(g0)(0) = D(g1)(0);**

$$eq7 := b0 = b1$$

> **eq8 := D(g1)(1) = D(g2)(1);**

$$eq8 := 2 a1 + b1 = 2 a2 + b2$$

The 2nd derivative equal at the first endpoint to zero

> **eq9 := (D@@2)(g0)(-2) = 0;**

$$eq9 := 2 a0 = 0$$

> **solve({eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9});**

$$\{a0 = 0, c0 = 3, c1 = 3, b0 = \frac{3}{2}, b1 = \frac{3}{2}, a1 = \frac{-7}{2}, c2 = \frac{19}{2}, a2 = 3, b2 = \frac{-23}{2}\}$$

**Question 3: According to the following table of data for g(x):**

x	g(x)
-2	0
0	3
1	1
3	2

Fit these data with cubic spline interpolation.

**SOLUTION using MAPLE 11**

Cubic Spline Interpolation:

> **g0 := x-> a0\*x^3 + b0\*x^2 + c0\*x + d0;**

$$g0 := x \rightarrow a0 x^3 + b0 x^2 + c0 x + d0$$

> **g1 := x-> a1\*x^3 + b1\*x^2 + c1\*x + d1;**

$$g1 := x \rightarrow a1 x^3 + b1 x^2 + c1 x + d1$$

> **g2 := x-> a2\*x^3 + b2\*x^2 + c2\*x + d2;**

$$g2 := x \rightarrow a2 x^3 + b2 x^2 + c2 x + d2$$

The 1st & 2nd functions equal at the interior knots

> **eq1 := g0(0)=3;**

$$eq1 := d0 = 3$$

> **eq2 := g1(0)=3;**

$$eq2 := d1 = 3$$

The 2nd & 3rd functions equal at the interior knots

> **eq3 := g1(1)=1;**

$$eq3 := a1 + b1 + c1 + d1 = 1$$

> **eq4 := g2(1)=1;**

$$eq4 := a2 + b2 + c2 + d2 = 1$$

The 1st & 3rd functions equal at endpoints

> **eq5 := g0(-2)=0;**

$$eq5 := -8 a0 + 4 b0 - 2 c0 + d0 = 0$$

> **eq6 := g2(3)=2;**

$$eq6 := 27 a2 + 9 b2 + 3 c2 + d2 = 2$$

The 1st derivative equal at the interior knots

> **eq7 := D(g0)(0) = D(g1)(0);**

$$eq7 := c0 = c1$$

> **eq8 := D(g1)(1) = D(g2)(1);**

$$eq8 := 3 a1 + 2 b1 + c1 = 3 a2 + 2 b2 + c2$$

The 2nd derivative equal at the first endpoint and second end point to zero

> **eq9 := (D@@2)(g0)(-2) = 0;**

$$eq9 := -12 a0 + 2 b0 = 0$$

> **eq10 := (D@@2)(g2)(3) = 0;**

$$eq10 := 18 a2 + 2 b2 = 0$$

> **solve ({eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10});**  

$$\{ d0 = 3, d1 = 3, b0 = 6 a0, b2 = -9 a2, b1 = -\frac{19}{2} - 8 a2 - 16 a0, a1 = 6 + 8 a2 + 8 a0, \\ d2 = -15 a2 + \frac{1}{2}, c2 = 23 a2 + \frac{1}{2}, c1 = 8 a0 + \frac{3}{2}, c0 = 8 a0 + \frac{3}{2}, a2 = a2, a0 = a0 \}$$

**GOOD LUCK!**

**STUDY + LEARN + SOLVE QUIZ 4 = PASS QUIZ 4**

Best Regards

Mohammed K.A Kaabar

Website: <http://www.mohammed-kaabar.net>