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* Definition: Assume that $f$ and $g$ are differentiable in some open interval $I$ containing $a$, except possibly at $a$, and $g'(x) \neq 0$ for all $x \in I$.

If $\lim_{x \to a} \frac{f(x)}{g(x)}$ is an indeterminate form of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ provided that the limit on the right hand side exists or is $\infty$.

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Ex 1 Find $\lim_{x \to 0} \frac{1-\cos x}{x}$.

$\frac{1-1}{0} = \boxed{\frac{0}{0}}$

Solution:

$$\lim_{x \to 0} \frac{1-\cos x}{x} = \lim_{x \to 0} \frac{\sin x}{1} = \sin(0) = \boxed{0}$$

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Ex 2 Find $\lim_{x \to 0} \frac{e^x - 1}{2x}$

$\frac{e^0 - 1}{0} = \frac{1-1}{0} = \boxed{\frac{0}{0}}$

Solution:

$$\lim_{x \to 0} \frac{e^x - 1}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{e^0}{2} = \boxed{\frac{1}{2}}$$

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Ex3] Find $\lim_{x\to\infty} \frac{\ln x}{x}$. $\frac{\infty}{\infty}$

Solution: $\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \left(\frac{1}{x}\cdot 1\right) =$

$= \frac{1}{\infty} = \boxed{0}$

Ex4] Find $\lim_{x\to 1} \frac{x^{100}-1}{x-1}$. $\frac{1-1}{1-1} = \frac{0}{0}$

Solution: $\lim_{x\to 1} \frac{x^{100}-1}{x-1} = \lim_{x\to 1} \frac{100x^{99}}{1} = 100(1)^{99} = \boxed{100}$

Ex5] Find $\lim_{x\to\infty} \frac{x^3}{e^{2x}}$. $\frac{\infty}{\infty}$

Solution:

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{x^3}{e^{2x}} = \lim_{x\to\infty} \frac{3x^2}{2e^{2x}} = \lim_{x\to\infty} \frac{6x}{4e^{2x}} = \lim_{x\to\infty} \frac{6}{8e^{2x}} =$

$= \frac{6}{8(\infty)} = \frac{6}{\infty} = \boxed{0}$