Calc 1 Review
Def's, Prop's and Thm's

Limits:

* Provided that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist and f is continuous at g(a) then

i. $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

ii. $\lim_{x \to a}\left[f(x) \begin{smallmatrix}\bullet\\ \div\end{smallmatrix} g(x)\right] = \lim_{x \to a} f(x) \begin{smallmatrix}\bullet\\ \div\end{smallmatrix} \lim_{x \to a} g(x)$

iii. $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$

* A Function f is said to be continuous at a point c if $\lim_{x \to c} f(x) = c$

* (Squeeze Thm.) Let f,g and h be functions such that $g(x) \leq f(x) \leq h(x)$ $\forall$x in an interval containing a.
If $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$
then $\lim_{x \to a} f(x) = L$

* (Derivative) Given that the following limits exist then

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = f'(x) = \lim_{h \to o} \frac{f(x+h) - f(x)}{h} = \lim_{x \to h} \frac{f(x) - f(h)}{x - h}$$

* (L'Hopitals Thm) Given the functions f and g differentiable on an open interval about a point c and

$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = \infty$ **or** 0 then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Derivatives:

* Power Rule: $\frac{\mathrm{d}}{\mathrm{d}x} x^r = r x^{r-1}$

* Sum/Difference Rule: $\frac{\mathrm{d}}{\mathrm{d}x}[f(x) \pm g(x)] = \frac{\mathrm{d}}{\mathrm{d}x} f(x) \pm \frac{\mathrm{d}}{\mathrm{d}x} g(x)$

* Product Rule: $\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

* Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$

* Chain Rule: $\frac{d}{dx}[f(g(x))]=\left[\frac{d}{du}f(u)\right]\left[\frac{du}{dx}\right]$ $where\ u=g(x)$

$$=f'(g(x))g'(x)$$

* Trig Derivatives:

i) $\frac{d}{dx}[\sin(x)]$ = cos(x), $\frac{d}{dx}\left[\sin^{-1}(x)\right]=\frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}[\cos(x)]$ = -sin(x), $\frac{d}{dx}\left[\cos^{-1}(x)\right]=\frac{-1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}[\tan(x)]=\sec(x)^2$, $\frac{d}{dx}\left[\tan^{-1}(x)\right]=\frac{1}{1+x^2}$

iv) $\frac{d}{dx}[\cot(x)]=-\csc(x)^2$, $\frac{d}{dx}\left[\cot^{-1}(x)\right]=\frac{-1}{1+x^2}$

v) $\frac{d}{dx}[\sec(x)]$ = sec(x)tan(x)

vi) $\frac{d}{dx}[\csc(x)]=-\csc(x)\cot(x)$

* Special Derivatives

i) $\frac{d}{dx}\ln(x)=\frac{1}{x}$

ii) $\frac{d}{dx}e^x=e^x$

iii) $\frac{d}{dx}b^x=b^x\ln(b)$

Integrals:

* $\int cf(x)dx = c\int f(x)dx \quad \forall\ c\epsilon\mathbb{R}$

* $\int[f(x)\pm g(x)]dx=\int f(x)dx\pm\int g(x)dx$

* If $F'(x) = f(x)$ **then** $\int f(x)dx = F(x) + c$

*(Substitution)

$$\int f(u)\left(\frac{du}{dx}\right)dx = \int f(u)du$$

*Special forms:

i) $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$

ii) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$

iii) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\left|\frac{u}{a}\right|\right) + c$

* $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$

* $\displaystyle\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where f is integrable on [a,b] and c ϵ(a,b)

*(Fundamental Thm of Calculus pt 1&2) Given f continuous on [a,b] and F is the antiderivative of f on [a,b] then,

i) $\displaystyle\int_a^b f(x)\mathrm{d}x$ = F(b) - F(a)

ii) $\displaystyle\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

* (Substitution for Definite Ints) Given f is continous on an interval contain g(a) and g(b) then,

$\displaystyle\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$ where u = g(x) which is continuous on [a,b]

Examples:

1.) $\lim_{x \to 0} x^{-1}\sin(8x) = 8$ (Hint: L'Hopitals)

2.) $\lim_{x \to 0} x^2\sin\left(\frac{1}{x}\right) = 0$ (Hint: Squeeze Thm)

3.) $\frac{d}{dx}\sqrt{3x + \frac{2}{\sqrt{x}} - \frac{5}{3x^3}} = \frac{\left(3 - x^{-\frac{3}{2}} + \frac{5}{x^4}\right)\left(3x + \frac{2}{\sqrt{x}} - \frac{5}{3x^3}\right)^{-\frac{1}{2}}}{2}$

4.) $\frac{d}{dx}\left(\arctan(7x^3)\right) = \frac{21x^2}{1 + 49x^6}$

5.) $\frac{d}{dx}\left[\frac{\ln(3x)}{e^{4x}}\right] = \frac{x^{-1} - 4\ln(3x)}{e^{4x}}$

6 $\frac{d}{dx}\ln\left(\frac{x^2 - \pi}{x^3 + 2}\right) = \frac{2x}{x^2 - \pi} - \frac{3x^2}{x^3 + 2}$ (Hint: use quotient rule for logs first)

7.) $\int \frac{1}{\sqrt{2 - x^2}}\, dx = \arcsin\left(\frac{\sqrt{2}\,x}{2}\right) + c$

8.) $\int x^{\frac{-4}{3}}\sqrt{11 - x^{\frac{-1}{3}}}\, dx = 2\left(11 - x^{-\frac{1}{3}}\right)^{\frac{3}{2}} + c$

9.) $\frac{d}{dx}\int_0^x e^{-3t^2}\, dt = e^{-3x^2}$ (Hint: FTOC)

10.) $\int (1 + \cos(3x))^5 \sin(3x)\, dx = \frac{-(1 + \cos(3x))^6}{18} + c$ (Hint: use substitution)

11.) $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{\ln(2T + 1)}{2T + 1}\, dT = \frac{3\ln(2)^2}{2}$ (Hint: use substitution)