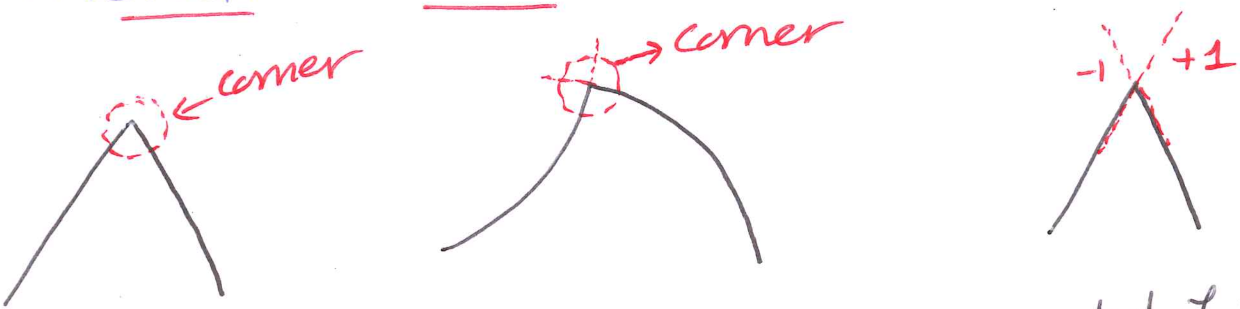
Mohammed KA Kaabar   Derivative (Part II)   Feb. 10th, 2016

* The four cases when the function does not have a derivative at a point.

I. Corner:

The left and right hand derivatives differ.

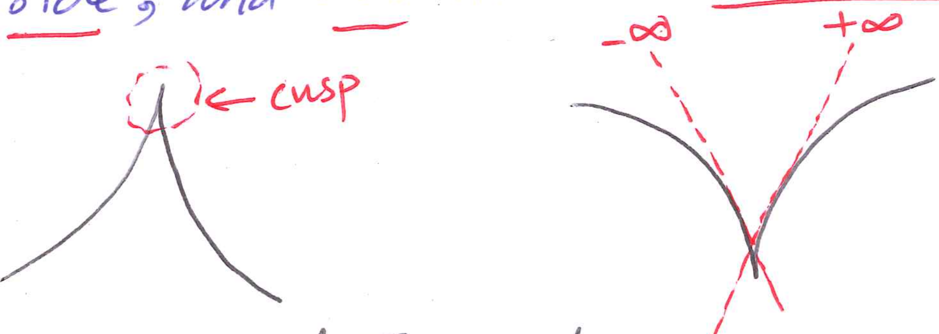


Two tangent lines

no derivative because they are different but if they are equivalent, then there is a derivative.

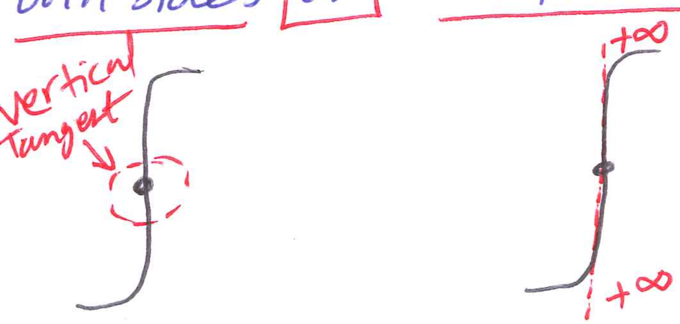
II. Cusp:

The slope of the secant line approaches $+\infty$ from one side, and $-\infty$ from the other side.



III. Vertical Tangent:

The slope of the secant line approaches $+\infty$ from both sides $\boxed{\text{or}}$ $-\infty$ from both sides.



IV. Discountinuity :

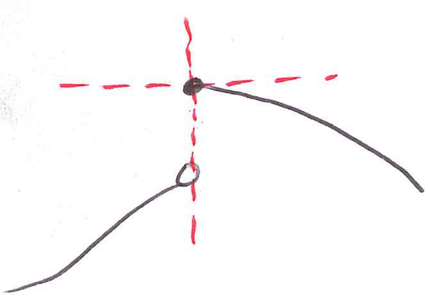
a. If f(x) has a derivative at a, then f(x) is continuous at a.

b. If f(x) is not continuous at a, then f(x) has no derivative at a.

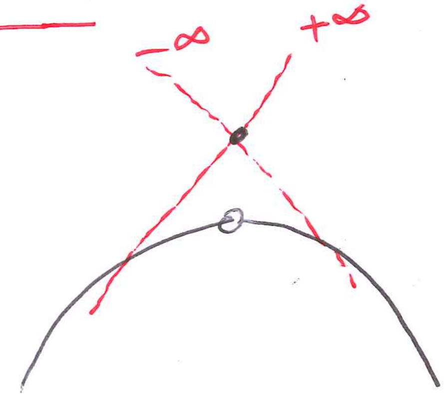
$$\underset{\text{derivative}}{p} \longrightarrow \underset{\text{continuous}}{q}$$

$$\underset{\text{Not continuous}}{\sim q} \longrightarrow \underset{\text{No Derivative}}{\sim p}$$

$$\sim p \longrightarrow \sim q \quad \times$$ Wrong Statement

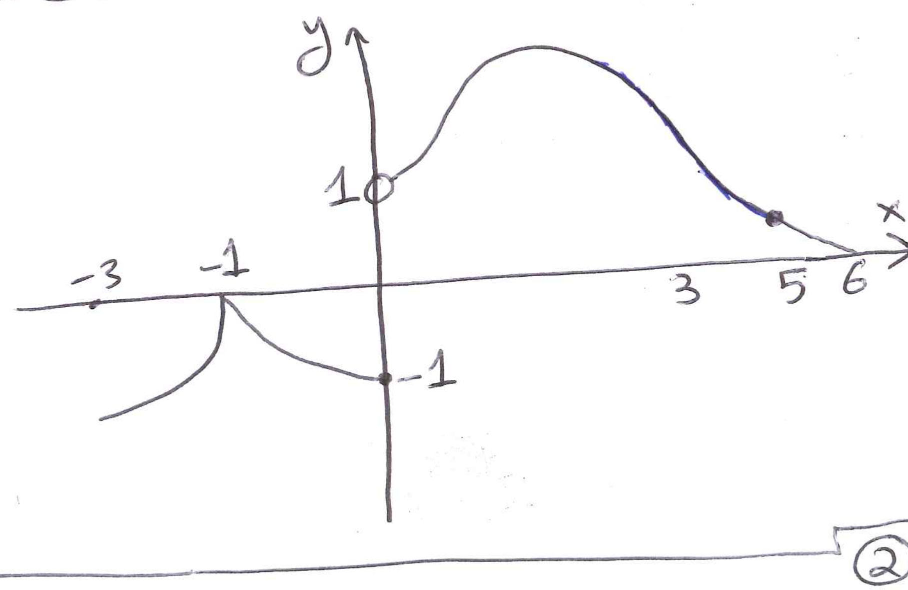


2 slopes



2 slopes

Ex 1 Find the points where there is no derivative:



Solution

f(x) has no derivative at :

x = -1 cusp

x = -3 and x = 6 End Points

x = 0 discontinuity

x = 5 Corner

x = 3 Vertical Tangent

* Differentiation Rules:

① $(c)' = 0$

② $(x^n)' = n x^{n-1}$

③ $(c f(x))' = c f'(x)$

④ $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

⑤ Product Rule: $(f(x) \cdot g(x))' = f'(x) g(x) + g'(x) f(x)$

⑥ Quotient Rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2}$

⑦ Chain Rule: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

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* Differentiation Theorems:

① $(e^x)' = e^x$

② $(\ln x)' = \frac{1}{x}$

Generally, $\boxed{\begin{array}{l} (e^{\square})' = e^{\square} \cdot \square' \\ (\ln \square)' = \frac{1}{\square} \cdot \square' \end{array}}$

③ $y = x^x \Rightarrow$ Take ln of both sides, we obtain:
$\ln y = \ln x^x \Rightarrow \ln y = x \ln x \Rightarrow$ Take the derivative of both sides $\Rightarrow \frac{1}{y} \cdot y' = \ln x + \frac{x}{x} \Rightarrow \frac{y'}{y} = 1 + \ln x \Rightarrow y' = y(1 + \ln x)$

$\boxed{y' = x^x(1 + \ln x)}$

Ex 2 Find the derivative for $(f \cdot g \cdot h \cdot m)$.

Solution:

$(f \cdot g \cdot h \cdot m)' = f'ghm + fg'hm + fgh'm + fghm'$

Ex 3 Find y' for the following:

a. $y = \frac{1}{2x} + \frac{1}{x\sqrt[3]{x}} + \pi$

b. $y = (x^2+1)(2x^3 - x + 7)$

c. $y = \frac{2x-3}{x^2+7}$

Solution:

Part a: $y = \frac{1}{2}x^{-1} + x^{-4/3} + \pi$

$y' = \frac{-1}{2}x^{-2} - \frac{4}{3}x^{-7/3} + 0$

$\boxed{y' = \frac{-1}{2}x^{-2} - \frac{4}{3}x^{-7/3}}$

Part b: $\boxed{y' = (x^2+1)(6x^2-1) + (2x^3-x+7)(2x)}$

Part c: $\boxed{y' = \frac{(x^2+7)(2) - (2x-3)(2x)}{(x^2+7)^2}}$