

* Definite Integral:

$$\int_a^b f(x) dx = F(b) - F(a)$$

Ex 1 Find $\int_0^1 x^2 dx$.

Solution:

$$\int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{(1)^3}{3} - \frac{(0)^3}{3} = \frac{1}{3} - \frac{0}{3} = \frac{1}{3} - 0 = \frac{1}{3}$$

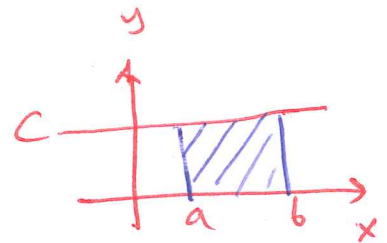
* Properties of the Definite Integral:

① $\int_a^a f(x) dx = 0$

② $\int_a^b f(x) dx = - \int_b^a f(x) dx$

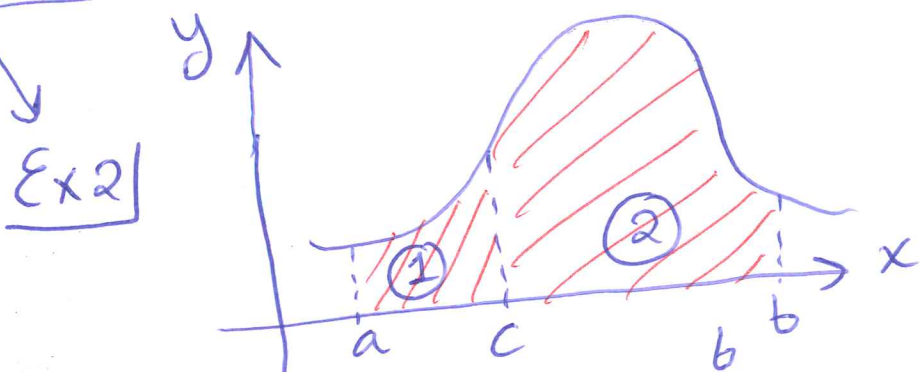
③ $\int_a^b c dx = c \int_a^b 1 dx = c x \Big|_a^b = c(b-a)$

④ $\int_a^b k f(x) dx = k \int_a^b f(x) dx$



⑤ $\int_a^b (\underline{f(x)} \oplus \underline{g(x)}) dx = \int_a^b f(x) dx \oplus \int_a^b g(x) dx$

⑥ $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$



$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

Ex 3] Given: $\int_{-1}^4 f(x) dx = 5$ & $\int_{-1}^3 f(x) dx = 10$

Find each of the following integrals:

① $\int_3^{-1} f(x) dx = - \int_{-1}^3 f(x) dx = \boxed{-10}$

② $\int_4^3 f(x) dx = \int_4^{-1} f(x) dx + \int_{-1}^3 f(x) dx = (-5) + (10) = \boxed{5}$

③ $\int_{-1}^4 (10f(x) + 3) dx = 10 \int_{-1}^4 f(x) dx + 3 \int_{-1}^4 1 dx = 10(5) + 3x \Big|_{-1}^4$
 $= 10(5) + 3(4 - (-1))$
 $= 50 + 3(5) = 50 + 15 = \boxed{65}$