Name: ______________________

Section: ______________________

Practice with Integrals Notes

Review:

- **Useful Integrals and Identities**

$$\int \sec^2(x)dx = \tan(x) + C \quad \bullet \quad \int \sec(x)\tan(x)dx = \sec(x) + C$$

$$\int \cosh(x)dx = \sinh(x) + C \quad \bullet \quad \int \sinh(x)dx = \cosh(x) + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \bullet \quad \int \frac{1}{x^2 + a^2}dx = \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \cot(x)dx = \ln|\sin(x)| + C \quad \bullet \quad \int \tan(x)dx = \ln|\sec(x)| + C$$

$$\int a^x dx = \frac{a^x}{\ln(a)} + C \quad \bullet \quad \int \frac{1}{x}dx = \ln|x| + C$$

$$\int \tan(x)dx = \ln|\sec(x)| + C \quad \bullet \quad \int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x)) \quad \bullet \quad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) + \cos^2(x) = 1 \quad \bullet \quad 1 - \sin^2(x) = \cos^2(x)$$

$$1 + \tan^2(x) = \sec^2(x) \quad \bullet \quad \sec^2(x) - 1 = \tan^2(x)$$

- **The Substitution Rule (u substitution using $u = g(x)$).**

  Indefinite

$$\int f(g(x))\, g'(x)dx = \int f(u)du$$

  Definite

$$\int_a^b f(g(x))\, g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

  Parameters: f must be continuous on the range of $u = g(x)$ <u>and</u> either g'(x) is continuous on the interval [a,b] (Definite Case) or g(x) is differentiable (Indefinite Case).

- **Integration by Parts** "Reversing the product rule"

  Let f and g be differentiable functions, then the product rule states

  $\frac{d}{dx}[f(x)g(x)] =$ ______________________

  Integrating both sides and splitting the integral implies:

  $f(x)g(x) =$ ______________________

  If we let $u = f(x)$, and $v = g(x)$ then $du = f'(x)dx$, and $dv = g'(x)dx$. With a little rearranging we get

$$\boxed{\int udv = uv - \int vdu}$$