Sec 7.1 #59, sec 7.2 #55 **HW 2 SOLUTION** 5/5

Written Assignment #2 Colleen Gibbons

Sec 7.1 (59) Area of a region between curves

Find area of region bounded by curves

$Y = \frac{x^2}{x^3-3x}$ and $y = \frac{1}{x^3-3x}$ on the interval $[2,4]$

$$A = \int_2^4 \frac{x^2}{x^3-3x} - \frac{1}{x^3-3x}\,dx = \int_2^4 \frac{x^2-1}{x^3-3x}\,dx$$

$u = x^3-3x \quad du = 3x^2-3$

$\frac{1}{3}du = x^2-1\,dx$

$x=4,\ 4^3-12=52$

$x=2,\ 2^3-6=2$

$$\frac{1}{3}\int_2^{52} \frac{1}{u}\,du$$

$$\hookrightarrow \frac{1}{3}\ln u\Big|_2^{52} \to \frac{1}{3}(\ln 52 - \ln 2) = \frac{1}{3}\ln\frac{52}{2}$$

$$= \frac{\ln(26)}{3}$$

Sec 7.2 (55) Logarithm base b: Prove that

$$\int \log_b x\,dx = \frac{1}{\ln b}(x\ln x - x) + C$$

$$= \int \frac{\ln x}{\ln b}\,dx \to \frac{1}{\ln b}\int \ln(x)\,dx$$

integration by parts: $uv - \int v\,du$

$$= \frac{1}{\ln(b)}\left(\ln(x)\,x - \int \frac{1}{x}\,x\,dx\right)$$

$$\hookrightarrow = \frac{1}{\ln(b)}\left(x\ln(x) - \int 1\,dx\right)$$

$$= \frac{x\ln(x) - x}{\ln(b)} = \frac{1}{\ln b}(x\ln(x) - x)$$