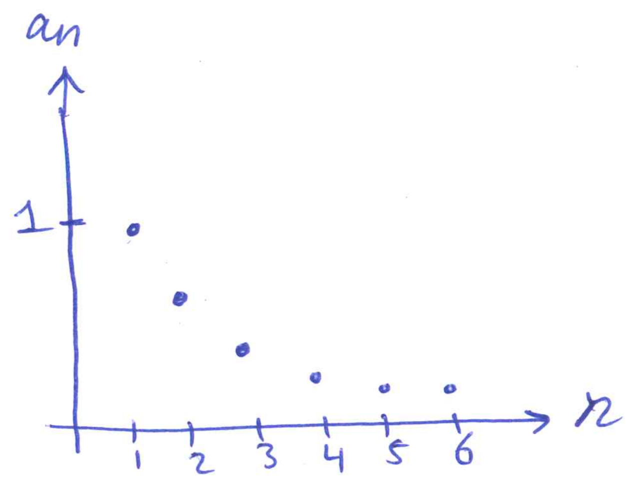
* Sequences:

$$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$$

$a_1 \quad a_2 \quad a_3 \quad a_4$



$\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$

$a_n = \frac{1}{n^2}$, $\lim\limits_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty} = 0$ converges.

Example 1: $1, -1, 1, -1, \ldots$

$a_n = (-1)^{n+1}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} (-1)^{n+1} = \infty$ diverges.

Example 2: $1, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \frac{-1}{6}, \ldots$

$a_n = (-1)^{n+1} \cdot \frac{1}{n} = \frac{(-1)^{n+1}}{n}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{(-1)^{n+1}}{n} = 0$ Converges.

Example 3: $\left\{\frac{n}{n+1}\right\}_1^{\infty}$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$\lim\limits_{n\to\infty} \frac{n}{n+1} = \lim\limits_{n\to\infty} \frac{\cancel{n}}{\cancel{n}} = 1$ converges

* Geometric Sequences and Series:

Example 4: $2+6+18+54+162+\cdots$

geometric series $r=\frac{6}{2}=\boxed{3}$

Example 5: $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$

geometric series $\boxed{r=\frac{1}{2}}$

$$S_n = a+\cancel{ar}+\cancel{ar^2}+\cdots+\cancel{ar^{n-1}}$$

$$- \quad rS_n = \cancel{ar}+\cancel{ar^2}+\cancel{ar^3}+\cdots+ar^n$$

$$S_n - rS_n = a + ar^n$$

$$\Rightarrow S_n(1-r) = a(1-r^n) \Longrightarrow \boxed{S_n = \frac{a(1-r^n)}{(1-r)}}$$

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1<r<1 \\ \text{diverges} & \text{if } |r|>1 \end{cases}$$

*Theorem: the geometric series: $a+ar+ar^2+\cdots$ is convergent if $|r|<1$, and its sum is $\frac{a}{1-r}$. Otherwise, if $|r|>1$, then the geometric series diverges.