



Study Guide 1

MATH 172 Lab: Sections 7 and 8

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Note: This study guide contains my practice questions that I think will be useful for preparing you for the first exam in Calculus II.

Question 1: Evaluate the integral: $\int \frac{x^2}{1+x^6} dx$.

$$\int \frac{x^2}{1+(x^3)^2} dx$$

$$\int \frac{x^2}{1+(x^3)^2} dx = \int \frac{1}{1+u^2} \cdot \frac{1}{3} du = \frac{1}{3} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{3} \tan^{-1}(u) + C$$

Substitution: $u = x^3$
 $du = 3x^2 dx \Rightarrow x^2 dx = \frac{du}{3}$

$$\Rightarrow \frac{1}{3} \tan^{-1}(x^3) + C$$

Question 2: Evaluate the integral: $\int \frac{4}{x^2+2x+5} dx$.

By completing the square:

$$x^2 + 2x + 5 = [x^2 + 2x + 1 + 4]$$

$$\left(\frac{2}{2}\right)^2 = 1^2 = 1 = [(x+1)^2 + 4] = 4 \left[\frac{(x+1)^2}{4} + 1 \right]$$

$$= 4 \left[\left(\frac{x+1}{2}\right)^2 + 1 \right]$$

$$\int \frac{4}{x^2+2x+5} dx = 4 \int \frac{1}{x^2+2x+5} dx =$$

$$= \frac{4}{4} \int \frac{1}{\left(\frac{x+1}{2}\right)^2 + 1} dx = \int \frac{1}{\left(\frac{x+1}{2}\right)^2 + 1} dx$$

Question 3: Evaluate the integral: $\int \frac{4x}{x^2+4} dx$.

$$\Rightarrow \int \frac{4x}{x^2+4} dx = 2 \int \frac{2x}{x^2+4} dx$$

$$= 2 \ln|x^2+4| + C$$

Now, substitution:

$$u = \frac{x+1}{2} = \frac{1}{2}x + \frac{1}{2}$$

$$du = \frac{1}{2} dx \Rightarrow 2du = dx$$

Question 4: Evaluate the integral: $\int \cot(x) dx$.

$$\Rightarrow \int \cot(x) dx = \int \frac{\cos(x)}{\sin(x)} dx$$

$$= \ln|\sin(x)| + C$$

$$\Rightarrow \int \frac{1}{u^2+1} du = 2 \tan^{-1}(u) + C$$

$$= 2 \tan^{-1}\left(\frac{x+1}{2}\right) + C$$

Question 5: Evaluate the integral: $\int \frac{(\sin^{-1}(x))^3}{\sqrt{1-x^2}} dx$.

Substitution: $u = \sin^{-1}(x)$
 $du = \frac{1}{\sqrt{1-x^2}} dx$ $\Rightarrow \int u^3 du = \frac{u^4}{4} + C$
 $= \frac{(\sin^{-1}(x))^4}{4} + C$

Question 6: Evaluate the integral: $\int (e^{2x} + \frac{1}{\sqrt{x}}) dx$.

$$\int (e^{2x} + \frac{1}{\sqrt{x}}) dx = \int e^{2x} dx + \int \frac{1}{x^{1/2}} dx = \int e^{2x} dx + \int x^{-1/2} dx$$

$$= \frac{e^{2x}}{2} + \frac{x^{1/2}}{1/2} + C$$

Question 7: Evaluate the integral: $\int \sec(x) dx$.

$$\int \sec(x) dx = \int \frac{\sec(x)(\sec(x) + \tan(x))}{\sec(x) + \tan(x)} dx = \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x) + \tan(x)} dx$$

$$= \ln|\sec(x) + \tan(x)| + C$$

Question 8: Evaluate the integral: $\int \frac{\cos(x)}{\sqrt[3]{4\sin(x)+1}} dx$.

Substitution: $u = 4\sin(x) + 1$
 $du = 4\cos(x) dx \Rightarrow \cos(x) dx = \frac{du}{4}$
 $\Rightarrow \int \frac{\cos(x) dx}{\sqrt[3]{4\sin(x)+1}} = \frac{1}{4} \int \frac{1}{\sqrt[3]{u}} du = \frac{1}{4} \int u^{-1/3} du = \frac{1}{4} \int u^{-1/3} du = \frac{u^{2/3}}{4(2/3)} + C$
 $= \frac{1}{4} \left(\frac{3}{2}\right) (4\sin(x)+1)^{2/3} + C$

Question 9: Find the area bounded by $y = 3x - x^2$ and the x -axis from $x = -1$ to $x = 4$.

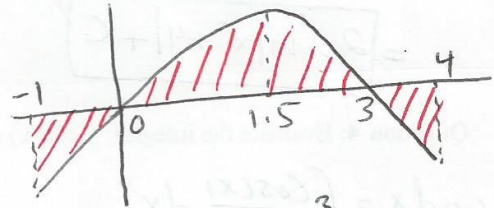
$$y = 3x - x^2$$

$$y' = 3 - 2x$$

$$y'' = -2$$

Find critical points $y' \begin{matrix} \nearrow \\ +++ \\ \rightarrow \end{matrix}$ $\begin{matrix} \rightarrow \\ --- \\ \searrow \end{matrix}$
 1.5

$$y' = 3 - 2x = 0 \Rightarrow x = \frac{3}{2} = 1.5$$



$$A = \int_{-1}^0 -(3x - x^2) dx + \int_0^3 (3x - x^2) dx + \int_3^4 -(3x - x^2) dx$$

$$= \frac{11}{6} + \frac{9}{2} + \frac{11}{6} = \frac{49}{6}$$

Question 10: Let R be a region bounded by $y = x^2$; and $y = 3x$.

Find the volume of the above region generated by revolving R .

Part a: About $y = 9$.

Washer

$$V = \int_0^3 [\pi(9-x^2)^2 - \pi(9-3x)^2] dx$$

OR Shell

$$V = \int_0^9 2\pi(9-y)(\sqrt{y} - \frac{y}{3}) dy$$

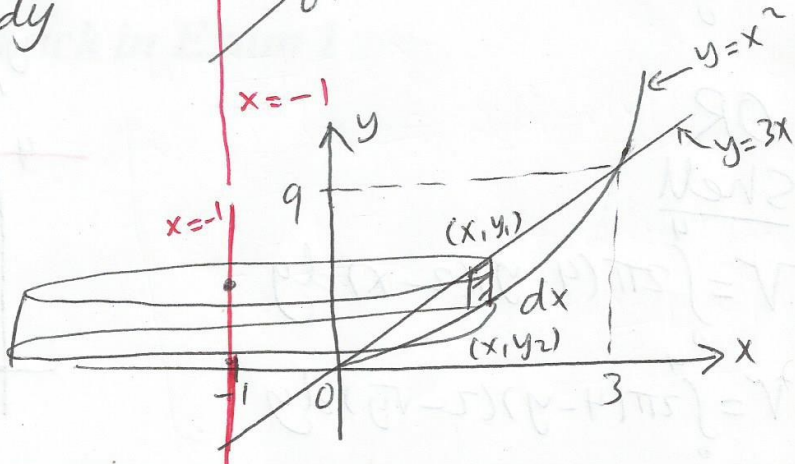
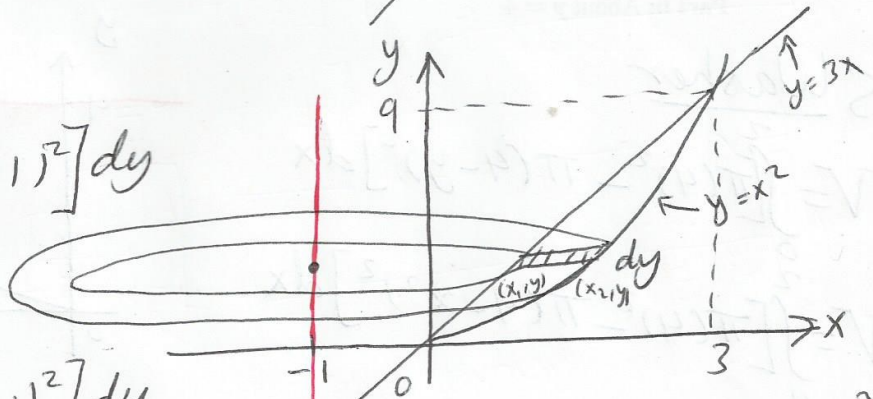
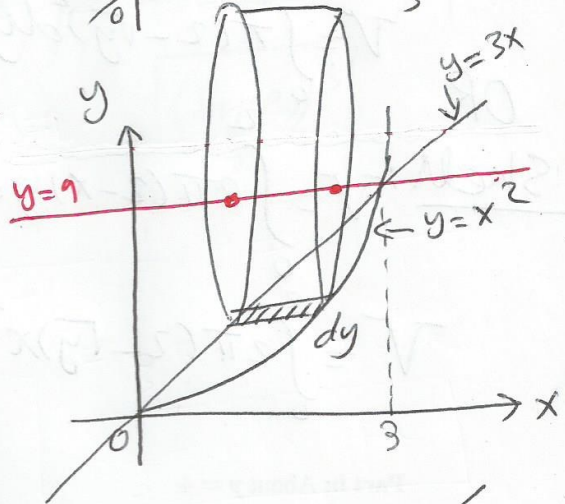
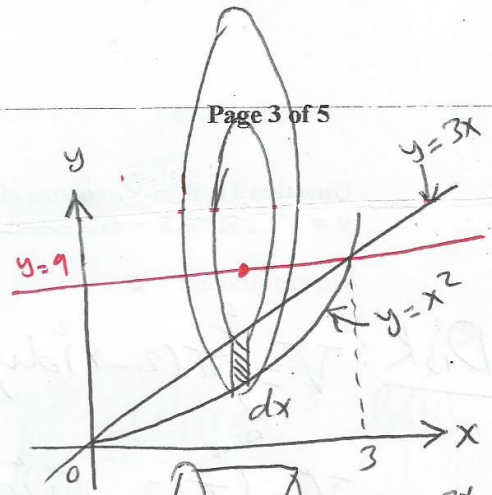
Part b: About $x = -1$.

$$V = \int_0^9 [\pi(x_2+1)^2 - \pi(x_1+1)^2] dy$$

$$= \int_0^9 [\pi(\sqrt{y}+1)^2 - \pi(\frac{y}{3}+1)^2] dy$$

OR Shell

$$V = \int_0^3 2\pi(x+1)(y_1 - y_2) dx$$



Question 11: Find the volume of the solid generated by revolving the region bounded by $y = x^2$, and the x -axis from $x = 0$ to $x = 2$.

Part a: About $x = 2$.

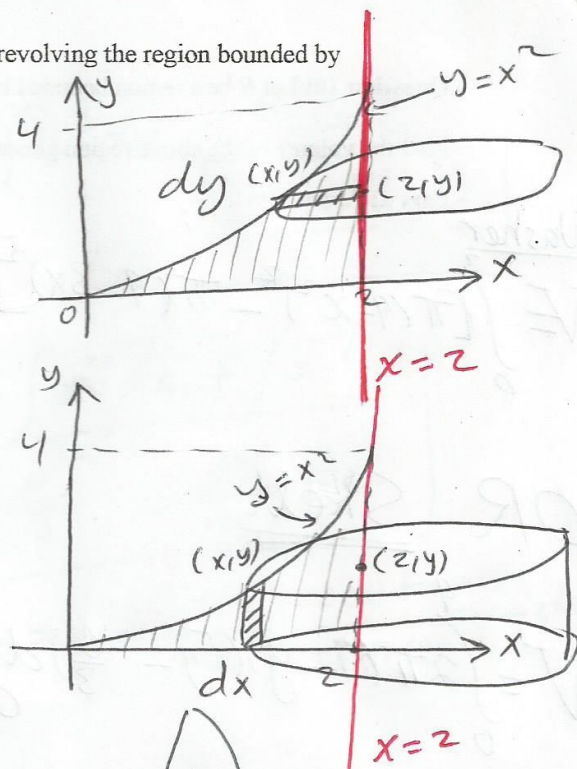
Disk: $V = \int_0^4 \pi(2-x)^2 dy$

$V = \int_0^4 \pi(2-\sqrt{y})^2 dy$

OR

Shell: $V = \int_0^2 2\pi(2-x)y dx$

$V = \int_0^2 2\pi(2-x)^2 dx$



Part b: About $y = 4$.

Washer

$V = \int_0^2 [\pi(4)^2 - \pi(4-y)^2] dx$

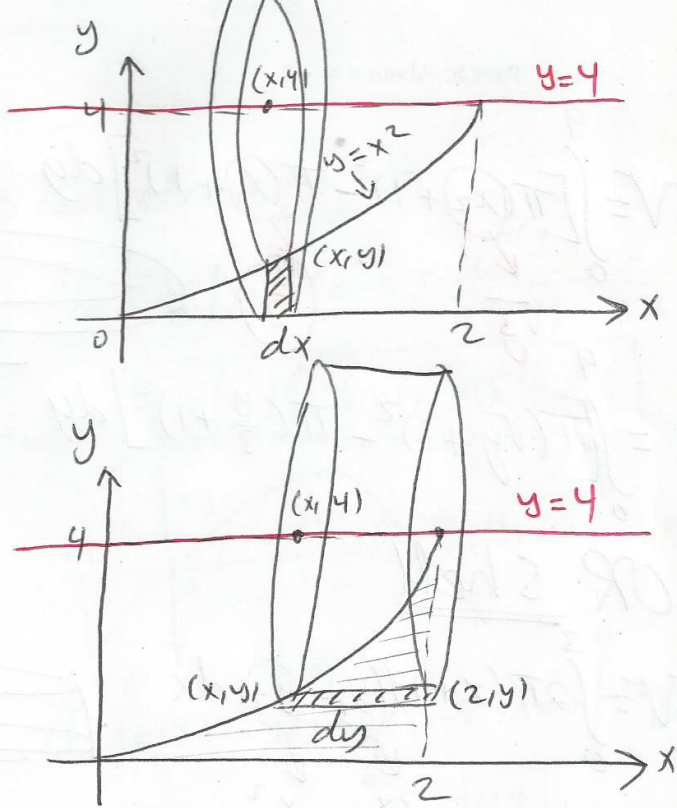
$V = \int_0^2 [\pi(4)^2 - \pi(4-x^2)^2] dx$

OR

Shell

$V = \int_0^4 2\pi(4-y)(2-x) dy$

$V = \int_0^4 2\pi(4-y)(2-\sqrt{y}) dy$



Question 12: Evaluate the integral: $\int_1^e \frac{(\ln(x))^2}{x} dx$.

Substitution: $u = \ln(x)$

$$du = \frac{1}{x} dx$$

$$\begin{aligned} \Rightarrow \int (u)^2 du &= \frac{u^3}{3} + C \Rightarrow \frac{(\ln(x))^3}{3} \Big|_1^e = \frac{(\ln(e))^3}{3} - \frac{\ln(1)}{3} \\ &= \frac{(\ln(e))^3}{3} \end{aligned}$$

Good Luck in Exam 1