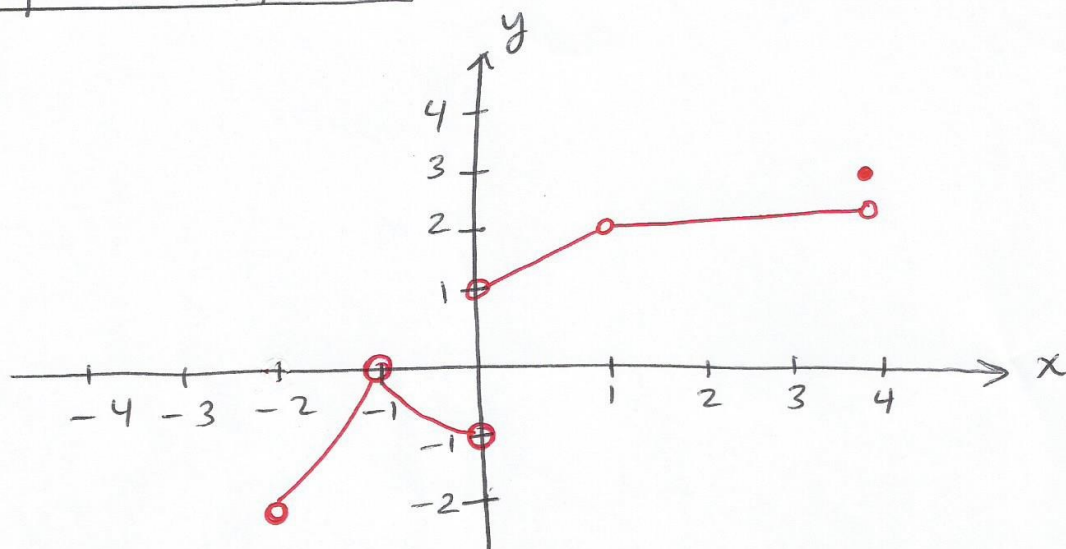


* Graphical Approach:



Find the following:

① Domain: $(-2, 4] \setminus \{-1, 0, 1\}$

② Range: $(-2, 0) \cup (1, 2] \cup \{3\}$

↳ Note: Only look at y-axis.

③ $\lim_{x \rightarrow 0} f(x) =$ Does NOT Exist (DNE)

↳ $\lim_{x \rightarrow 0^+} f(x) = 1$
 ↳ $\lim_{x \rightarrow 0^-} f(x) = -1$
 So, it's DNE.

④ $\lim_{x \rightarrow 1} f(x) = 2$

⑥ $\lim_{x \rightarrow -2^+} f(x) = -2$

⑤ $\lim_{x \rightarrow 3} f(x) = 2$

⑦ $\lim_{x \rightarrow -2} f(x) = \text{DNE}$

⑧ $\lim_{x \rightarrow -1} f(x) = 0$

Ex1 Find $\lim_{x \rightarrow 0} \frac{1}{x} = ?$

Solution:

$$\lim_{x \rightarrow 0} \frac{1}{x} = \frac{1}{0} = \infty$$

x	1/x
0.1	10
0.01	100
0.001	1,000
0.000001	1,000,000
	↓
	$+\infty$

Notes:	
$\frac{1}{0}$	$\pm\infty$
$\frac{1}{0^+}$	$+\infty$
$\frac{1}{0^-}$	$-\infty$

Ex2 Find $\lim_{x \rightarrow 3^+} \frac{x}{3-x} = ?$

Solution:

$$\lim_{x \rightarrow 3^+} \frac{x}{3-x} = \frac{3}{0^-} = -\infty$$

Ex3 Find $\lim_{x \rightarrow 1} \frac{2x}{|x-1|} = ?$

Solution:

$$\lim_{x \rightarrow 1} \frac{2x}{|x-1|} = \frac{2}{|1-1|} = \frac{2}{0^+} = +\infty$$

Ex4 Find $\lim_{x \rightarrow 0} e^{-1/x^2} = ?$

Solution: $\lim_{x \rightarrow 0} e^{-1/x^2} = e^{-1/0} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

* Theorem: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \boxed{1}$

Ex5) Find $\lim_{x \rightarrow 0} \frac{x}{\sin x} = ?$

Solution:

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} = \frac{1}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{1}{1} = \boxed{1}$$

Ex6) Find $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x} = ?$

Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{x}}{\frac{\sin 5x}{x}} = \frac{\lim_{x \rightarrow 0} \frac{\sin 3x}{x}}{\lim_{x \rightarrow 0} \frac{\sin 5x}{x}} = \frac{3}{5} = \boxed{\frac{3}{5}}$$

Divide
Both numerator
and denominator
by x

Ex7) Find $\lim_{x \rightarrow 0} \frac{3x^2}{x \sin x + \sin x} = ?$

Solution:

We divide both numerator and denominator by x^2 .

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{3x^2}{x \sin x + \sin x} &= \lim_{x \rightarrow 0} \frac{\frac{3x^2}{x^2}}{\frac{x \sin x}{x^2} + \frac{\sin x}{x^2}} = \\ &= \lim_{x \rightarrow 0} \frac{3}{\frac{\sin x}{x} + \frac{\sin x}{x^2}} = \frac{3}{1+1} = \boxed{\frac{3}{2}} \end{aligned}$$

Ex 8 Find $\lim_{x \rightarrow 0} x^2 \csc^2 x = ?$

$$\csc x = \frac{1}{\sin x}$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} x^2 \csc^2 x &= \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{x \cdot x}{\sin x \cdot \sin x} = \\ &= \lim_{x \rightarrow 0} \left[\left(\frac{x}{\sin x} \right) \cdot \left(\frac{x}{\sin x} \right) \right] = \left(\lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} \right) \cdot \left(\lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} \right) = 1 \cdot 1 = \boxed{1} \end{aligned}$$

Ex 9 Find $\lim_{x \rightarrow 2} \frac{|x-2|}{4-2x} = ?$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 2^+} \frac{x-2}{4-2x} &= \frac{\overset{-1}{\cancel{x-2}}}{2(2-\cancel{x})} = \frac{-1}{2} \\ \lim_{x \rightarrow 2^-} \frac{-(x-2)}{4-2x} &= \frac{-\overset{-1}{\cancel{x-2}}}{2(2-\cancel{x})} = \frac{1}{2} \end{aligned} \Rightarrow \text{DNE}$$

Ex 10 Find $\lim_{x \rightarrow 0} = ?$ for $f(x) = \begin{cases} x+2, & x < 0 \\ 10, & x = 0 \\ \cos x, & x > 0 \end{cases}$

defined-piecewise-Functia.

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \cos x = \cos 0 = 1 \\ \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} (x+2) = 0+2 = 2 \end{aligned} \Rightarrow \text{DNE}$$