

Activity: Differential Activity

Directions and Overview

This activity will introduce differential equations, describe a model of infusion of drugs into the bloodstream, and introduce the solution of differential equations. Please do the numbered activities within the text below. Try to be neat and organized. Justify your reasoning, and remember to show your work.

Introduction

A **Differential Equation**, or DE, is an equation involving derivatives of a function. An Ordinary Differential equation, or ODE, has only one independent variable.

Some generic examples of ODEs are given below:

$$y' = ky \tag{1}$$

$$y' = ky + b \tag{2}$$

$$y'' = ky \tag{3}$$

$$y'' + ty' = 1 \tag{4}$$

$$y' = ky(M - y) \tag{5}$$

In these examples $y' = \frac{dy}{dt}$, $y'' = \frac{d^2y}{dt^2}$, and $y(t)$ is the function with independent variable t . In a typical problem the function y is initially unknown, so it is sometimes referred to as the 'unknown function'. Of course, different symbols could be used for the independent variable and the unknown function. The 'order' of a differential equation is the order of the highest derivative in the differential equation.

Recall that a linear function of x has the form $mx + b$, where m and b do not depend on x . A linear function of two variables x and y has the form $ax + by + c$ where a, b and c do not depend on x or y . A general 'linear' first order ODE has the form $ay + by' + c = 0$, where a, b and c do not depend on y or y' but may depend on t . An ODE that is not linear is said to be 'nonlinear'.

I₁: For each of the examples 1 \rightarrow 5 given above, determine the order of the ODE and whether it is linear or nonlinear, and why. Assume that k, b and M do not depend on y .

Solution: